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A GOAL PROGRAMMING APPROACH FOR DETERMINING THE FORCE STRUCTURE OF NAVAL SURFACE GROUPS USING THE ANALYTIC HIERARCHY PROCESS

by

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A GOAL PROGRAMMING APPROACH FOR DETERMINING THE FORCE STRUCTURE OF NAVAL SURFACE GROUPS USING THE ANALYTIC HIERARCHY PROCESS

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ABSTRACT

A methodology for determining the force structure of naval surface groups is developed. A survey of naval surface officers is used to determine a surface ship's relative superiority over the others with respect to several factors (e.g., speed, warfare capabilities, surveillance capabilities, and fuel consumption). The Analytic Hierarchy Process (AHP) is employed to convert survey judgments into numerical preference weights. The AHP weights are then used as objective function coefficients in the mixed integer goal programming model formulations. The object of each model formulation is to select a preferred mix of ship types by minimizing the total deviation from one or more force level goals given certain system constraints such as budget, weapon requirements, and/or existing force levels.

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LIST OF ABBREVIATIONS

AAW	Anti Air Warfare
AHP	Analytic Hierarchy Process
AMPW	Amphibious Warfare
ASM	Anti Submarine Missile
ASUW	Anti Surface Warfare
ASW	Anti Submarine Warfare
CG	Cruiser
CIWS	Close-in Weapon System
DDG	Guided Missile Destroyer
FFG	Guided Missile Frigate
FSG	Guided Missile Corvettes
GAMS	General Algebraic Modeling System
GP	Goal Programming
MAUT	Multi-attribute Utility Theorem
MIP	Mixed Integer Problem
PBFA	Guided Missile Fast Patrol Boat
POS	Protection of Shipping
R&D	Research and Development
SAM	Surface to Air Missile
SSM	Surface to Surface Missile
TNSG	Turkish Naval Surface Group

EXECUTIVE SUMMARY

The design of a modern military force structure is a complicated process. This process involves many competing elements. Political pressures, budget constraints and foreign threats are some of the main effects in force structuring design. Currently, there are several different methodologies for determining the force structure of a naval unit. This thesis proposes an alternative methodology for determining the force structure of the Turkish Naval Surface Group (TNSG).

The TNSG force structuring problem (selecting constituent ship types for this surface group) can be modeled as a project selection problem. Project selection models are examined, and it is considered that the model must be capable of solving multi-objective problems (decreasing the cost and increasing the effectiveness of the force for our study).

Two main models are formulated to solve this force structuring problem. Model 1 is formulated to determine the number and mix of ship types to purchase or build given a constant budget and weighted preferences for each ship type. In this model, the main goal is to maximize the effectiveness of the force mix when a constant budget is given. Model 2's formulation uses the weapon requirements needed to meet the threat's specific capabilities (defined in the mission areas of air, surface, submarine, and amphibious operations), ship weapon capacities for these requirements, and the weighted preferences for the ship types. In second model, the main goal is to minimize the cost of providing a fixed force effectiveness.

There are additional factors, aside from cost and weapon requirements, which must be included when considering the mix and size of the naval surface group. These factors should include not only the ship attributes such as speed, cost, warfare capabilities, and fuel consumption, but with respect to these attributes, the relative advantage held by each ship in accomplishing the mission. A survey is a method to obtain subjective judgments rating ships with respect to the above factors. The survey judgments are

converted into numeric values, using the Analytic Hierarchy Process which are then used as objective function coefficient weights in our model formulations.

The scenario on which weapon requirements are based is a conventional war situation with a specific country. Exact values for future weapon requirements will be determined by the Turkish Naval Surface Group Staff after examining the force structure and weapon capabilities of the threat countries. In this study, however, only approximate values are used for requirements and budgets. Hence, this model is a generic model and can be used by any country by changing these requirement and budget values.

This study also provides a sensitivity analysis of the changes in the force mix with changes in the future weapon requirements and budget values. It also provides a comparison of the models that are used and selects the best model for determining the best mix of ship types.

Several computer packages and programs are used to solve these force structuring problems. With the use of software and computer support, it is easy to make changes in the model values. The optimum ship mixes for the models are found using these computer programs. The comparison of the models is made according to the smallest goal deviation and the highest benefit-cost ratio, and it is found that Model 2 (minimum cost, fixed weapon requirements) gives the best ship mix for TNSG.

I. INTRODUCTION

A. BACKGROUND

The design of a modern military force structure is a complicated process. This process involves many competing elements. Currently, force structure design is affected by political pressures, budget constraints and foreign threats. This process impacts on Turkey's ability to protect its national interests in peacetime and war. The main purpose of this study is to propose a model which can generate a solution to the Turkish Naval Surface Group's (TNSG) force structuring problem.

The mission of TNSG is to conduct Anti Surface Warfare (ASUW) operations, Anti Submarine Warfare (ASW) operations, Anti Air Warfare (AAW) operations, and Amphibious Warfare (AMPW) operations in peacetime (for training) and in war. The current TNSG inventory contains only Guided Missile Destroyers (DDG), Guided Missile Frigates (FFG), and Guided Missile Fast Patrol Boats (PBFA). However there have been some proposals to add other types of surface ships (Cruisers and Corvettes) to the inventory.

Modern cruisers (CG) and guided missile destroyers (DDG) operate in support of carrier battle groups, surface action groups, amphibious groups, and replenishment groups. Both CGs and DDGs are multi-mission (ASW, AAW, and ASUW) surface combatants; however, the CG is also capable of operating independently and is capable of serving as a flagship of a surface action group. While guided missile frigates (FFG) can fulfill the Protection of Shipping (POS) mission as ASW combatants for amphibious expeditionary forces, underway replenishment groups and merchant convoys, they can also perform AAW and ASUW missions. But their weapon capacity is limited when compared with CGs and DDGs. Figure 1.1 shows an Oliver Hazard Perry class frigate that the Turkish Navy is planning to buy from U.S. Navy. Corvettes (FSG) are also multi-mission (ASW and ASUW) surface combatants. New types of corvettes also have AAW capabilities, with the addition of surface to air guided missiles (SAM). Guided missile fast

patrol boats (PBFA) are single mission (ASUW) surface combatants. Because of their fast speed and addition of the surface to surface guided missiles (SAM), PBFAs are more effective in surface warfare. PBFAs also operate in coastal patrol and interdiction surveillance.

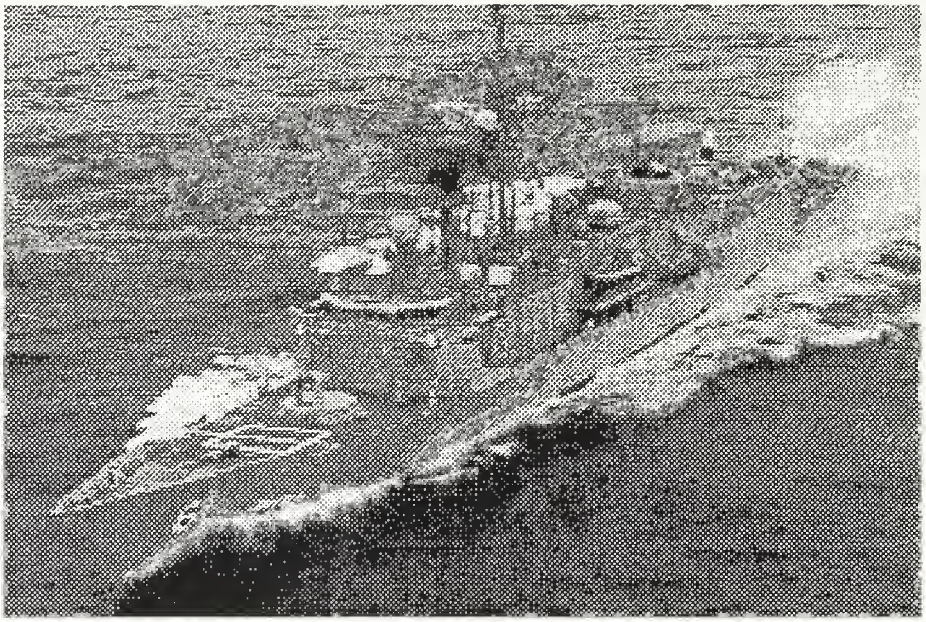

Figure 1.1 Guided Missile Frigate (FFG) .

Important factors in the TNSG's ability to conduct their own missions and to assist other forces in conducting operations are the ship-type characteristics of speed, ASUW capabilities (both open sea and shoreline), ASW capabilities, AAW and Close-in Weapon Systems (CIWS) capabilities (against guided missiles and aircraft), AMPW capabilities, and surveillance (target reporting) capabilities. These characteristics change according to the ship types utilized for a mission. Table 1.1 summarizes surface ship characteristics which will be used in our proposed model. Ship characteristics are taken from Jane's Fighting Ships 1996/1997 edition [Ref. 1]. Ship cost is certainly an important consideration to take into account when planning within a specified budget. The cost values shown in Table 1.1 are taken from Healy [Ref. 2].

Ship (Class)	Cost ($\$10^9$)	Speed (kts)	SSM (launcher)	SAM (launcher)	ASM (launcher)	GUNS Short R.	Long R.
CG Ticon.	0.90	30	16	8	14	2	2
DDG Spruance	0.60	33	8	8	11	2	2
FFG Meko	0.30	32	8	8	6	0	1
FSG Type 420	0.20	32	8	4	6	1	1
PBFA Dogan	0.13	41	8	--	--	2	--

Table 1.1 Surface Ships Characteristics.

B. OBJECTIVE

The TNSG inventory contains old ships that have been bought from other countries and new modern ships. The main goal is to modernize all of the ships in TNSG. Current modernization progress is mainly focused on building FFGs and PBFAs. The objective of this thesis is to provide an alternative methodology for determining the force structure of the Turkish Naval Surface Group in this modernization effort. The problem is to determine the best mix of ship types given some constraints, such as budgets and/or specific weapon requirements.

The model proposed here obtains survey judgments which rate each surface ship type's relative superiority over the others with respect to selected characteristics (e.g., speed, warfare capabilities, surveillance capabilities, and fuel consumption) which enable the ship's operations in a warfare area. The Analytic Hierarchy Process (AHP) is used to convert these subjective judgments into numeric values that are used as objective function coefficient weights in several goal programming model formulations. The object of each formulation is to select the best mix of surface ships by minimizing the total deviation from

one or more force level goals given certain system constraints, such as budgetary requirements, weapon requirements, and/or existing force levels.

C. SCOPE

The scenario on which weapon requirements are based is a conventional war situation with a specific country. Exact values for future weapon requirements will be determined by the Turkish Naval Surface Group Staff by examining the force structure and weapon capabilities of the threat countries. In this study, however, only approximate values are used for requirements and budgets. Hence, this model is a generic model and can be used by any country by changing these requirements and budgets.

This paper is organized to logically discuss the modeling process and results. Chapter II will consist of a search for the most appropriate project selection model among all such models currently used in the operations research literature. Chapter III will contain the model development process, and Chapter IV will examine the formulation of the models being implemented. Chapter V will present the computer programs used to exercise the model as well as model results and output. Chapter VI will contain the sensitivity analysis of the models in various situations. The final chapter will present conclusions and recommendations.

II. LITERATURE REVIEW

The force structuring of the Turkish Naval Surface Group (selecting constituent ship types) can be viewed as a project selection problem. The intent of this chapter is to provide a brief overview and assessment of the literature in the area of project selection. The four basic types of project selection models that will be discussed below are subjective models, financial analysis models, risk analysis models, and mathematical programming models.

A. SUBJECTIVE MODELS

The subjective models are the simplest form of formal R&D project evaluation. The checklist and scoring models are the subjective models that are used most frequently. Liberatore and Titus [Ref. 3] reported the results of an empirical study on the usage of quantitative techniques for R&D project management in 40 respondents from 29 random Fortune 500 industrial firms. They found that almost half of the respondents had used checklist or scoring models in their R&D project funding process.

The checklist approach involves the completion of profile charts for each project that is considered for funding. A list of criteria is first set to develop a profile. Then the candidate projects which meet these criteria or requirements are rated on a subjective scale such as high/medium/low or favorable/neutral/unfavorable. These ratings can be done by a committee or a single individual. The opinions of single individuals or committees can be summarized in a checklist by averaging their opinions. Figure 2.1 presents an example of a checklist.

The advantage of a checklist lies in its simplicity and ease of use. A checklist can also provide a pictorial display of a project's merits and limitations. Non-economical factors that are awkward or nearly impossible to include in more formal model constructions, such as social impacts and environmental concerns, can be added to a checklist without any difficulty.

	Criteria								
	Profitability			Marketability			Success likelihood		
Projects	3	2	1	3	2	1	3	2	1	Total Score
Project A	x				x			x		7
Project B		x		x					x	6
Project C			x			x			x	3

Figure 2.1 Illustration of a Checklist For Three Projects [Ref. 4].

While the simplicity and the ease of the checklist model is very appealing, it can also be dangerous, since complex problems may be overlooked. Complicated relationships are not easily incorporated into checklists. Although many important factors may be included in a checklist, the relevance or importance of each individual factor is not captured.

Scoring models attempt to remedy this problem by assigning weights to individual criteria and summarizing results as a single project score. Each candidate project is scored on each criterion, using an appropriate scoring scale (e.g., 10=excellent, 1=unacceptable). Each criterion is weighted relative to its importance. These scores and weights are combined according to the following model:

$$T_i = \sum_j w_j * s_{ij} \quad .$$

Here, T_i is the overall project score for project i, w_j is the relative criterion weight for the j^{th} criterion, and s_{ij} is the criterion score for i^{th} project on the j^{th} criterion. Figure 2.2 presents an example of a scoring model.

Criteria (i)	Criterion Weight (w_j)	*	Project Score (s_{ij})	=	Criterion Score (w_i*s_{ij})
Probability of Success	4		5		20
Profitability	3		10		30
Cost	2		6		12
Patentability Rating	1		3		3
					$T_i = 65$

Figure 2.2 Illustration of a Scoring Model [Ref. 4].

After finding the overall score for each project, the projects are prioritized from highest to lowest score. The highest scores are the most preferable to the decision maker. The weights and scores can be derived by using several methods. These include rank-ordering of attributes, pairwise comparisons, and other various rankings. Moore and Baker [Ref. 5] demonstrated that increasing the number of scoring intervals improves the accuracy of the scoring models. However, using psychometric testing, nine has been found as the maximum number of intervals that should be used [Ref. 5].

Moore and Baker [Ref. 6] compared scoring models with more sophisticated models such as economic, risk analysis, and constrained optimization models (mathematical programming models). Scoring models require less data input than the three other models. Because of data requirements, the scoring models have been viewed as most applicable during the initial stage of research, while the other model types are more appropriate for advanced research and engineering development analyses. They concluded that scoring models were not as constrained or weak as the three other models.

Scoring models retain the advantage of checklists and profile charts in terms of their ability to consider a wide range of economic as well as non-economic criteria. Another advantage of the scoring model is the opportunity to use simple, low-cost methods for data acquisition. The scoring model also allows the decision maker to predetermine the impact of every factor while making his decision. Some examples of scoring model applications have been presented by Moore and Baker [Ref. 6], Dean and Nishry [Ref. 7], and Motley and Newton [Ref. 8].

The major disadvantage of the scoring model is that the project score is dimensionless which limits its use to rank-order comparisons. Another problem of scoring models is that the model development is non-formal. The weights and subjective scores can be determined in a number of ways, so there can be some difficulties in developing them precisely across different raters.

B. FINANCIAL ANALYSIS MODELS

Liberatore and Titus [Ref. 3] reported that almost every firm in their study was familiar with financial project selection and funding techniques and 62% of the firms they studied used these techniques. The two major methods of financial modeling were Net Present Value (NPV)/Internal Rate of Return (IRR) and benefit/cost analysis models.

The IRR of a project can be defined as the effective rate of interest which will be earned on the money invested in the project. The NPV of a project is calculated by subtracting its present discounted cost from its present discounted benefit. A positive NPV shows that initial and future costs of project are less than the benefits of project, and it is desirable. The projects with the highest IRR or NPV are preferred.

Benefit cost ratio models attempt to measure an estimated benefit or return from a project consistent with cost. Costs and benefit values are quantified in discounted dollar values. Costs are total resource costs of supporting the research project and benefits are the net earnings to be realized from the project. If the benefit/cost ratio of a project is greater than or equal to one, we can accept that project; otherwise, we can ignore it.

Benefit cost ratios can be easily expanded to include a wide range of considerations such that a number of risk factors which reduce the expected project benefits. These risk factors can be shown as the probability of success of the project at various stages of development. Jackson [Ref. 9] presented an example of a benefit/cost ratio model, Olsen's model, as:

$$V = \frac{r * d * m * s * p * n}{\text{Total Project Cost}}$$

where V is the economic value of the project, s is the annual sales volume derived from the project if the project succeeds, p is the profit per unit, and r, d, and m are the probabilities of research, development, and marketing success, respectively. The product's expected life span is represented by n. Souder [Ref. 10] presented another index model, Ansoff's model:

$$\text{Figure of merit} = \frac{r * d * m * (T + B) * E}{\text{Total Project Cost}}$$

where r, d, and m are the same as Olsen's model. T and B are subjective ratings of the technical and business merits of the project, respectively, and E is the present value of the earnings expected if the project succeeds.

The above benefit/cost ratio models do not include other subjective (non-economic) benefits or costs, such as social, environmental or political effects. These benefits and costs can be entered into the model by expressing them in dollar values like the other factors.

Benefit/cost ratio models are desirable since they overcome the dimensionless problem of checklist models and scoring models. There are, however, definite shortcomings of this model. One problem with this type of model is that the measures of expected values (inputs) are very difficult to obtain. The probabilities and cost and benefit estimations are hard to measure and require specific experience on the subject. It is also hard to express many non-economic factors in dollar terms. The benefits of military project selection are especially difficult to express in dollar terms.

Another problem with benefit/cost ratio models is that the benefit cost ratios are not a useful tool for evaluating the consequences of alternative funding levels. Each element of benefits in the ratio must be reevaluated if the funding level is reduced or increased. Finally benefit/cost ratio models do not recognize resource constraints.

C. RISK ASSESMENT MODELS

One of the more useful tools used in R&D project selection and funding is the risk assessment technique. Liberatore and Titus [Ref. 3] found that the most of the respondents in their survey were familiar with risk assessment techniques, however only 35% of the respondents used these techniques in their project selection or funding process. The decision tree models and Monte Carlo simulation models (risk analysis models) are the two most important and applicable risk assessment techniques.

Decision tree methods attempt to project the chain of activities that will occur between the beginning and the completion of a project. Each of the steps along the chain contains estimated values of probabilities of success, costs, and returns. By this means it is possible to predict the expected profitability of alternative projects.

The decision tree structure consists of decision points or nodes, outcome nodes, and branches emanating to and from each node. Each branch or path has a certain outcome and risk associated with it. The decision tree model is built by constructing a pay-off matrix that contains all outcomes and probabilities. The optimum path is found by starting at the right-hand side of the tree and following an expected value algorithm folding back to the starting point. At each node the expected value is calculated for all the branches leaving that node, and the path with highest expected value is the one to be selected.

One of the main advantages of the decision tree models is that they are analytically simple and can be graphically presented. The graphical representation of decision trees helps to clarify the available strategies and the potential risks, regrets, and trade-off. It makes the decision tree an excellent communication tool when communicating with high-level managers.

The applicability of decision tree models in R&D project funding situations was demonstrated by Jackson [Ref. 9] and Raiffa [Ref. 11], and an example of decision tree models for project selection was presented by Flinn and Turban [Ref. 12]. The major disadvantage of decision trees is that the outcomes at each node are represented as a few discrete events rather than a continuous distribution of possible outcomes.

Models that depend on inputs that are influenced by chance or estimated with uncertainty are called stochastic. The stochastic procedure for evaluating density functions (the probability distribution of a project's outcome or the rate of return) has become known as the Monte Carlo simulation technique. Monte Carlo simulation was introduced by John von Neumann and Stanislaw Ulam when they both worked on the Manhattan Project at the Los Alomos National Laboratory.

Monte Carlo simulation is based on the decision tree model. The difference between these two models is that in Monte Carlo simulation each of the nodes is replaced with a probability distribution and this provides a stochastic decision tree. The probability distribution at each decision point will likely affect the project's outcome. The simulation model selects one random value from each of these decision point distributions and computes the rate of return for this particular combination of random values. Numerous sets of random values can be selected to show the alternative rates of return for each set to the decision maker.

The Monte Carlo simulation technique generally provides a more accurate description of the R&D decision process and offers a better basis for making project selection than other methods. The improvements in outcomes also bring a significant increase in information requirements. The benefits from the successful completion of the project must be assessed and the probability distributions for each unknown research project outcome must be estimated, and these processes are difficult and costly in most situations. Hespos and Strassman [Ref. 13] developed the most renowned application of the Monte Carlo simulation technique to a R&D project selection and funding problem.

The major disadvantage of these two risk assessment models is that both models do not deal with resource constraints. Like financial analysis models, these two models fail to allocate scarce resources among various research projects.

D. MATHEMATICAL PROGRAMMING MODELS

Mathematical programming models have been used to solve many resource allocation, project selection, and capital budgeting problems for three decades. Surprisingly, the Liberatore and Titus study [Ref. 3] found that although most of the respondents have familiarity with mathematical programming models, there was no usage of these models for R&D project selection in the firms that they surveyed. The types of mathematical programming that have been used in R&D project selection and funding are linear programming, non-linear programming, integer programming, and goal programming.

The linear programming techniques are concerned with the efficient use or allocation of limited resources among two or more activities or projects to meet desired objectives. Asher [Ref. 14] gives an example of a linear programming model in a R&D project funding problem. The general form of that example and other project funding models are shown below as:

$$\max imize \quad \sum CX \qquad (2-1)$$

$$subject\ to \quad \sum AX \leq B \qquad (2-2)$$

$$0 \leq X \leq 1 \qquad (2-3)$$

where X is an n-component vector representing the funding levels of projects, C is an n-component vector representing the contribution (net profitability) of various projects, A is an m x n matrix representing resource usage of the projects, and B is the particular resource levels. The range of X represents the funding levels of the projects and varies from zero to one. In many situations, projects are selected for full funding or they are not selected at all. In these situations, X can only take on values of zero or one.

While many functional relationships in a mathematical model may be linear in nature, some relationships realistically are nonlinear. Nonlinear programming models are similar to linear models; the difference between the two models is that the objective function, constraint equations or both are nonlinear. Tyler, Moore and Clayton [Ref. 13] give an excellent example of nonlinear programming approaches to the project funding problem.

Linear, integer, and nonlinear programming models are restricted to the establishment of only a single objective function. Most real world problems involve multiple goals. Goal programming, a modification and extension of linear programming, was introduced to solve this problem by Charnes and Cooper [Ref. 16] in 1961.

The goal programming approach allows us to meet a system of complex objectives rather than a single objective. Unlike linear programming, the goal programming objective function usually does not contain choice variables. Instead it contains the positive and negative deviation variables from the designated goal constraints. The objective function

tries to minimize these deviations, based on the relative importance (weights) assigned to them.

The formulation can be expressed as follows:

$$\text{minimize} \quad \sum W^+ d^+ + W^- d^- \qquad (2\text{-}4)$$

$$\textit{subject to} \quad \sum gX - d^+ + d^- = G \qquad (2\text{-}5)$$

$$\sum AX \le B \qquad (2\text{-}6)$$

$$d^+, d^-, X \ge 0 \qquad (2\text{-}7)$$

The variables d^- and d^+ represent the negative and positive deviations from the goal constraints. The goal constraint coefficients and goals are shown as g and G, respectively (equation 2-5). W^- and W^+ are nonnegative constants representing the relative weights to be assigned to the respective negative and positive deviation variables. Equations 2-6 and 2-7 are the same as linear programming constraints regarding resource constraints, resource availability, and nonnegativity.

The goal programming models are most appropriate for modeling multi-attribute decision problems. Most of the methodologies used to solve linear programming problems, like duality, sensitivity analysis, nonlinear programming, etc. will work on goal programming problems with minor revisions to the algorithms. Salvia and Ludwig [Ref. 17] describe a goal programming model at the Lord Corporation to solve the project funding problem. Schniederjans [Ref. 18] listed goal programming applications by categorizing them according to their subjects.

E. PREFERRED MODEL

In this chapter, the four general types of project funding models were discussed: subjective models, financial analysis models, risk assessment models, and mathematical programming problems. Though each model type is important and has appropriate applications, the goal programming model is most applicable model for determining the force structure of the TNSG. There are several reasons for using this model. The main reason for using the goal programming model is that it permits specification of multiple objectives or targets to be achieved. Goal programming also has a great deal of flexibility

that is lacking in the other models mentioned in this chapter. Finally, the goal programming algorithm allows the use of resource constraints. Ruefli [Ref. 19] gives an example of the goal programming approach in the U.S. Department of Defense to solve a force structuring problem similar to the TNSG problem.

III. MODEL DEVELOPMENT

The goal programming technique discussed in the last chapter requires objective function coefficients. The intent of this chapter is to determine a methodology for converting subjective judgments into objective function coefficients (weights) for the goal programming model. These coefficients or weights represent the importance of each ship type for each goal or constraint in the goal programming model formulation. The problem the decision maker faces is how to select a method for determining meaningful weights among the numerous available approaches.

A. TRADITIONAL METHODS OF QUANTIFYING SUBJECTIVE JUDGMENTS

Data are produced using some system of measurement. Stevens [Ref. 20] defines measurement as " the assignment of numbers to observations according to a set of rules." The numerical values assigned when measuring objects implies a scale of measurement. Stevens [Ref. 20] gives the four levels of measurement represented by the four types of scales: nominal, ordinal, interval, and ratio. A nominal scale is simply categorizing a set of data into mutually exclusive subclasses. In an ordinal scale, the numbers are assigned to the various instances of the property, so that the order of numbers corresponds to the order of magnitude of the instances. Interval scales have all the properties of an ordinal scale, and in addition the distances between any two numbers on the scale are equally spaced. A ratio scale has all the characteristics of an interval scale, but it also has a true zero point as its origin and has the property of proportionality.

These levels of measurement are used in the process of manipulating the numbers into a meaningful value of the object being measured. This value would then be used as an objective function coefficient for the goal programming model. The four main traditional methods of quantifying subjective judgments are presented below.

1. Numerical Rating Method

The numerical rating method is a very simple and direct method for quantifying subjective judgments. This method was first proposed by Stevens [Ref. 21] as a method of

obtaining comparative rankings in psychophysical experiments. Judges are asked to associate rated items with fixed reference points. This can be done by assigning numbers, or by plotting points on a continuous number line. Lodge [Ref. 22] gives an example of numerical rating that is used in obtaining opinions about how serious certain crimes are. The crime of a stolen car parked on the street was selected as a reference point and given a seriousness score of 100. Subjects were asked to rate other crimes using this reference point. If the judges think another crime is one tenth as less serious than the car theft, the seriousness score of that crime should be 10; if the crime was considered two times more serious, the score of that crime should be 200. After obtaining all the subjective responses, the numerical estimates are computed by taking their geometric mean. The continuous number line representation of this example is shown in Figure 3.1.

```
------------X---------------------X------------------------X------------
    less serious crime         car theft            more serious crime
```

Figure 3.1 Numerical Rating Continuous Line.

The primary advantage of the numeric rating method is computational simplicity. On the other hand, there are some disadvantages of this method; the main problem is that there is no natural origin for judgments, and judges frequently disagree with the reference point positioning. There are also no bounds on the interval scale; the lower bound (no crime situation) of the crime example is set to zero, but there is no upper bound.

2. Categorical Judgment Method

A commonly used means of obtaining numerical ratings from subjective ratings is the categorical judgment method, wherein judges assign instances to previously ordered categories. For example, program managers could be asked to rate a project milestone risk according to a scale of very low, low, average, high, or very high. The number of categories used can range from two to nine according to desires and resources of the surveyor and the skills of the raters [Ref. 23].

The procedure begins by rating the item in question and then constructing a matrix of cumulative frequency data with n row instances and m column categories. Each entry of

this matrix represents the number of judges who rated instance i in category j. A cumulative relative frequency matrix is developed from this matrix. In this matrix, entries are the proportions of the judges rating instance i in or below category j. The elements of this matrix are considered as areas under a standard normal curve and are converted to the corresponding z values. These values are then recorded in a z_{ij} matrix consisting of n rows and m-1 columns, since the rightmost column may be omitted for computational purposes. The row average, r_i, and column average, c_j, are calculated, and a grand average G is found by averaging the column averages. A column sum of squares is computed as shown below:

$$SSC = \sum_j \left(c_j - G\right)^2 \quad . \qquad (3\text{-}1)$$

Then for each row, the following is computed:

$$SSR_i = \sum_j \left(z_{ij} - r_i\right)^2 \quad . \qquad (3\text{-}2)$$

The scale value of the instances, s_i, are found by solving the following equation in each row:

$$s_i = G - r_i * \sqrt{SSC / SSR_i} \quad . \qquad (3\text{-}3)$$

The main disadvantage of the categorical judgment method is that it obtains values with interval scale properties that can be linearly transformed to any other scale. Although it is more sophisticated than the numerical rating method, it is still computationally easy. The major disadvantage of this method is that the precision of its results is limited by the number of categories selected for use in the survey [Ref. 23].

3. Least Squares Method

The least squares method was first proposed by Guilford [Ref. 24] as a means of obtaining scaled interval values from ordinal or comparative judgments. The inputs of this procedure are obtained by asking judges to do some ordinal ranking of various instances of a selected property.

The method is started by taking the responses of the judges comparing several items with respect to a particular characteristic. For example, a group of editors might be

asked to rate three word processing programs of different software companies in terms of the overall quality of the program. Suppose that an editor judges program B is better than program C, which is better than program A. These responses can be shown in a frequency matrix (Table 3.1).

f_{ij}	A	B	C
A		1	1
B			
C		1	

Table 3.1 Least Squares Method Scoring Matrix.

Since B is the preferred program by the judge, the entries are made in the corresponding rows of the column B that were rated inferior to B, which in this case are rows A and C. Since C was the second preferred program, an entry is made in the C column and A row.

The responses of all judges are recorded in this manner, and collected to a f_{ij} frequency matrix as in Table 3.2. The sum of the cross-diagonal elements of this matrix will be equal to the total number of the judges. For example, there were 100 judges in Table 3.3, and, as an example 54 judges A superior to C and 46 judges C superior to A

f_{ij}	A	B	C
A	--	28	46
B	72	--	65
C	54	35	--

Table 3.2 Least Squares Method Observed Frequency Matrix.

The next step is to convert the frequency matrix f_{ij} to a probability matrix P_{ij}. The P matrix can be calculated by using the following equation:

$$P_{ij} = \frac{f_{ij}}{f_{ij} + f_{ji}} \qquad (3\text{-}4)$$

For the above example, the probability matrix was obtained and is shown in Table 3.3.

p_{ij}	A	B	C
A	0.50	0.28	0.46
B	0.72	0.50	0.65
C	0.54	0.35	0.50

Table 3.3 Least Squares Method Probability Matrix.

The important thing in Table 3.3 is that the diagonal entries of the probability matrix are set equal to 0.5. The probability matrix is then converted to the standard normal matrix X by subtracting the mean value of 0.5 from each value of p_{ij} and dividing this differences by the standard deviation of p_{ij}. The X_{ij} values are the standard normal variables corresponding to the P_{ij} values of the probability matrix. In our example, the X_{ij} matrix is shown in Table 3.4. The least squares estimate of scale values s_j was obtained by taking the mean of each column in the matrix X. The least squares solution requires that all elements of matrix X be present, however, some entries of this matrix can be vacant. When probability matrix entries are equal to 1.00 or 0.00, we can not obtain corresponding X_{ij} entries. Torgerson [Ref. 25] stated that other least squares procedures have been developed for both this situation and the situation which judges do not rank all instances (incomplete probability matrices).

X_{ij}	A	B	C
A	0.00	-0.58	-0.10
B	0.58	0.00	-0.39
C	0.10	0.39	0.00
$\sum_i x_{ij}$	0.68	-0.19	-0.49
$s_j = \frac{1}{n}\sum_i x_{ij}$	0.23	-0.06	-0.17

Table 3.4 Least Squares Method Standard Normal Matrix.

As in the categorical judgment method, the least squares estimate of scale values are linearly transformable to other scales. The use of an ordinal rating scale requires less time and effort on the part of the judges than other methods, so the survey forms of this

method are simple to use. The judges can simply list the instances in the order of importance regarding the compared factor. The main disadvantage of this method is that it requires a large number of judges to produce a reasonably accurate probability matrix. The judges sometimes do not rank all instances for various reasons, and this makes the least squares procedure difficult to use in scale development.

4. Constant Sum Method

The constant sum method, developed by Comrey in 1950 [Ref. 26], quantifies subjective ratings using pairwise comparisons. In this method, each instance is compared with each other by splitting 100 points. There will be n(n-1)/2 pairs that must be considered, and 100 points will be divided between each in accordance with absolute ratio of the greater to the lesser. For example, if a judge gives 80 points to instance A and 20 points to instance B, this indicates that A is four times more important than B. In the same manner a split of 60-40 would indicate a ratio of three to two, and 50-50 that two instances have the same magnitude.

An example is used to illustrate this procedure. Suppose two judges are asked to evaluate three books on the basis of their content. Table 3.5 represents their respective comparison matrices where p_{ij} is the number of points is given to book i when compared

JUDGE 1				JUDGE 2			
p_{ij}	A	B	C	p_{ij}	A	B	C
A	50	20	30	A	50	30	40
B	80	50	60	B	70	50	80
C	70	40	50	C	60	20	50

Table 3.5 Constant Sum Method Comparison Matrix.

with book j. Both judges preferred book A to B, book A to C, and book C to B, but the intensities of these endorsements are different. The next step is to construct a matrix V by averaging the p_{ij} values across judges as shown in Table 3.6.

v_{ij}	A	B	C
A	50	25	35
B	75	50	70
C	65	30	50

Table 3.6 Constant Sum Method Average Comparison Matrix.

Another matrix W is formed from matrix V_{ij}. The w_{ij} values are computed using the following equation:

$$w_{ij} = v_{ij} / v_{ji} \quad . \qquad (3\text{-}5)$$

The W matrix for this example is shown in Table 3.7.

w_{ij}	A	B	C
A	1.00	0.33	0.54
B	3.00	1.00	2.33
C	1.86	0.43	1.00

Table 3.7 Constant Sum Method W Matrix.

The scale values can be computed by taking the n^{th} root of each column product, where n is the number of instances compared in our problem. In other words,

$$S_j = \left(\prod_i w_{ij} \right)^{1/n} . \qquad (3\text{-}6)$$

The calculation and results of our book example are demonstrated in Table 3.8.

$S_1 = [(1.00)*(3.00)*(1.86)]^{1/3} = 1.732$
$S_2 = [(0.33)*(1.00)*(0.43)]^{1/3} = 0.521$
$S_3 = [(0.54)*(2.33)*(1.00)]^{1/3} = 1.079$

Table 3.8 Constant Sum Method Scale Value Calculation.

The constant sum method provides quantitative values that allows linear transformations and arithmetic operations. The scale, therefore, presses ratio properties rather than interval scale properties. Consistency is one of the problems of this method

that occurs when the number of instances is large. The ability of judges must be greater in order to provide consistent ratings.

The traditional methods of quantifying subjective evaluation are not adequate for determining the force structure of the Turkish Naval Surface Group. The disadvantages of these methods indicate that another method that does not suffer from these disadvantages is necessary.

B. ANALYTIC HIERARCHY PROCESS (AHP)

The Analytic Hierarchy Process, a method for quantifying subjective variables, was developed and introduced by Professor Thomas L. Saaty of the Wharton School, University of Pennsylvania, and documented in his book of the same name [Ref. 27]. In the past few years AHP has been used in various decision-making models. Vargas [Ref. 28] lists over 25 specific applications of AHP.

The AHP procedure that has been popularized by Saaty contains four basic steps. These four steps in converting subjective judgments of a decision problem into numerical values are shown in Figure 3.2.

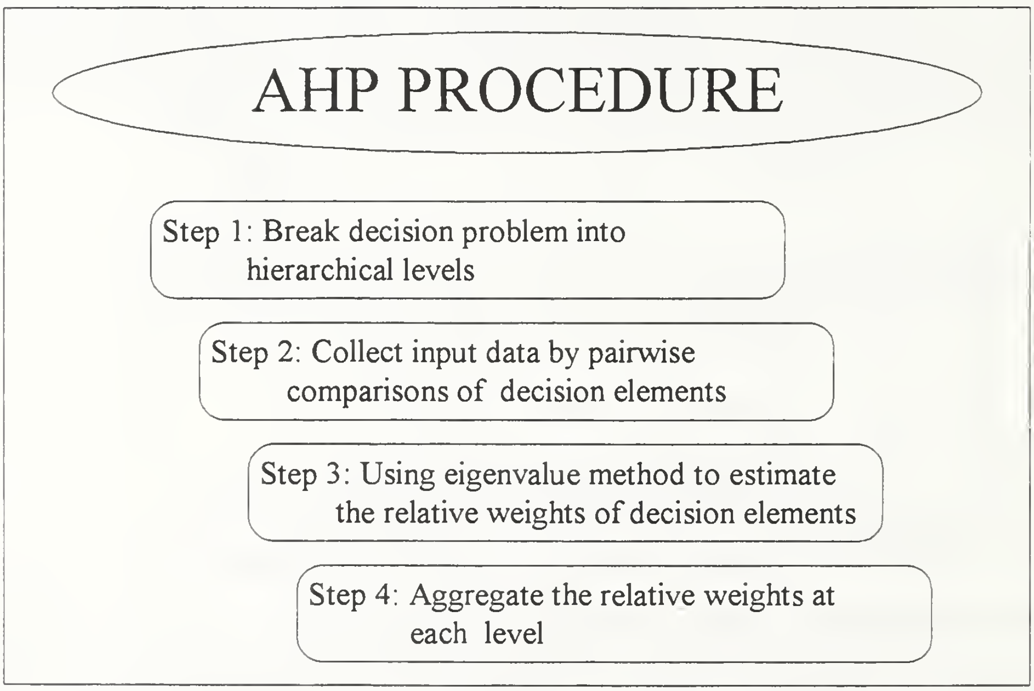

Figure 3.2 AHP Procedure.

The first step is to break down the problem into a hierarchy of decision elements. The decision maker must develop a logical representation of the factors and levels involved in the problem scenario. The top level of the hierarchy, called the *focus*, consists of only one element: the overall objective of the problem. The subsequent levels of hierarchy include more specific objectives, attributes, or factors necessary to achieve the overall objective. Details of these attributes increase at the lower levels of the hierarchy. The final level of hierarchy contains the specific decision alternatives.

The following car buying problem will be used to illustrate the AHP procedure in detail. For the decision to purchase a new automobile, the objective of buying a car is placed at the top of the hierarchy. Attributes of the car that influence the buying decision, such as cost, dependability, and comfort, are placed in the next level of the hierarchy, and the various car alternatives are listed at level three of Figure 3.3.

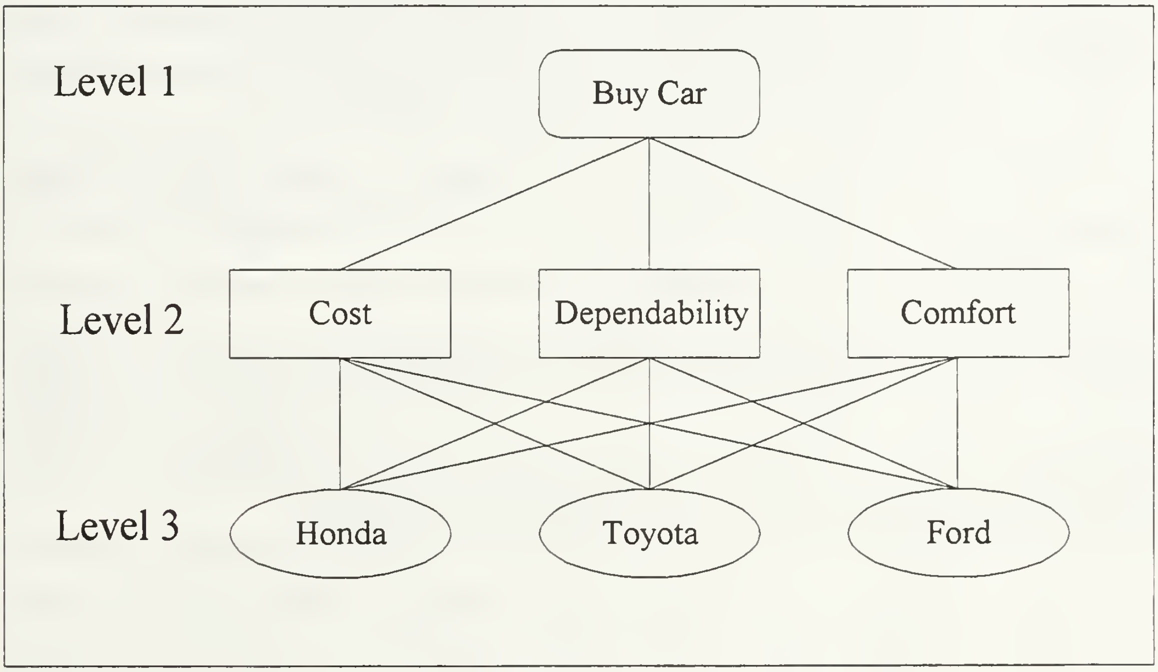

Figure 3.3 Car Buying Hierarchical Scheme.

In the second step, judges are asked to make pairwise comparisons of the factors of one level that contribute to achieving the objective of the next higher level using the pairwise comparison scale shown in Table 3.9.

Intensity of Importance	Definition	Explanation
1	Equal importance.	Two elements contribute equally to the property
3	Moderate importance of one over another.	Experience and judgment slightly favor one element over another
5	Essential or strong importance	Experience and judgment strongly favor one element over another
7	Very strong importance	An element is strongly favored and its dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of information
2,4,6,8	Intermediate values	Compromise is needed between two judgments
Reciprocals	When activity i compared to j is assigned one of the above numbers, then activity j compared to i is assigned its reciprocal.	
Rationals	Ratios arising from forcing consistency of judgments.	

Table 3.9 AHP Pairwise Comparison Scale [Ref. 29].

As in the constant sum method, each judge must make n(n-1)/2 pairwise comparisons (where n is the number of elements on a level of the hierarchy). Because of this need, Saaty has recommended keeping the number of elements in any level at no more than nine and the number of levels between three and five [Ref. 30]. For the car buying example, a comparison matrix in Table 3.10 was constructed to compare the three cars with respect to comfort. The table shows that, when compared to the Honda, the Toyota is one half as comfortable, and the Ford is one fourth as comfortable.

Comfort	Ford	Toyota	Honda
Ford	1	½	¼
Toyota	2	1	½
Honda	4	2	1

Table 3.10 AHP Car Example Comparison Matrix.

In step three, the relative weight of each element in a level is computed using the eigenvalue solution technique. These weights are found by normalizing the pairwise comparison matrix, summing over the rows and obtaining an average row sum. The row sums are the values of the priority vector (eigenvector). The priority vector of comfort for the car buying example is $(0.143^{1},0.286,0.571)^{T}$. These values state that the Honda is considered more comfortable than the other two cars, since the Honda comfort factor is twice the size of the Toyota and four times as large as the Ford value.

The "comfort" matrix used in the car buying example is consistent. In other words, the responses of the judges are not contradictory or conflicting (the Honda is preferred to the Toyota, the Toyota is preferred to the Ford, and the Honda is preferred to the Ford). An inconsistent matrix, for example, would show that while the Honda is preferred to the Toyota and the Toyota preferred to the Ford, the Ford would be preferred to the Honda. Complex decisions with more levels, attributes, and choices often contain inconsistencies. AHP measures the overall consistency of judgments by means of a consistency ratio (CR) that shows the quality of the data that has been input in the comparison matrices. The CR is computed by first finding the consistency index (CI). The consistency index is determined using the following equation:

$$CI = \frac{(\lambda_{\max} - n)}{(n-1)} \qquad (3\text{-}7)$$

where $\lambda_{\max}$ is the largest eigenvalue of the observed matrix of pairwise comparison and n is the number of the elements.

The consistency ratio can then be found by dividing the CI by a random index (RI). The RI are average consistency indices of randomly generated matrices whose reciprocal entries were taken at random from the values 1/9, 1/8, ..., 1, 2, ..., 8, 9. Saaty [Ref. 29] found that the value of the consistency ratio should be 10 percent or less. If it is more than 10 percent, the judgments are considered inconsistent. The consistency ratio for this example is zero.

---

[1] For example 0.143 is obtained by summing first row elements and dividing this value by the sum of all matrix elements.

Step four of the AHP aggregates the relative weights of the various levels from the previous step in order to produce a vector of composite weights. This vector is the weighted rankings of decision alternatives (or selection choices) with respect to the factor being studied. Step four starts at the top of the hierarchy and determines the weights at that level. These weights are then multiplied by the eigenvector at the next lower level obtaining new vectors. Repeating this procedure yields relative weights of the elements at the lowest level of the hierarchy.

Illustrating step four in our car buying example, the composite priority of the cars with respect to all the criteria is obtained by multiplying the priorities of the cars under each criterion by the priority of the criterion (in our example, the criteria in increasing order are: comfort, dependability, and cost[2]) and adding them across criteria. The calculation of the eigenvectors of cost and dependability are not shown. The procedure involved in step four is demonstrated in Table 3.11.

Step four of the car buying example shows that the Honda should be purchased based on the buyer's vehicle preferences regarding cost, dependability, and comfort. The Ford and the Toyota are essentially equivalent in preference.

Level 2 eigenvalues:	Cost = 0.5 Dependability = 0.3 Comfort = 0.2		
Level 3 eigenvalues			
Factor	Ford	Toyota	Honda
Cost	0.400	0.400	0.200
Dependability	0.200	0.100	0.700
Comfort	0.143	0.286	0.571
Ford weight : (0.5)(0.4)+(0.3)(0.2)+(0.143)(0.2) = 0.289 Toyota weight : (0.5)(0.4)+(0.3)(0.1)+(0.286)(0.3) = 0.316 Honda weight : (0.5)(0.2)+(0.3)(0.7)+(0.2)(0.571) = 0.424			

Table 3.11 Car Buying Example Step 4 Demonstration.

[2] The priority of criterions can be obtained by constructing a pairwise comparison matrix over criterions and calculating the weights in the same way shown in comfort example.

AHP has several advantages over the traditional methods of quantifying subjective judgments discussed earlier in this chapter. AHP requires the decision maker to logically structure a complex problem. AHP gives scaled values that are on a ratio scale, and AHP is the only method that provides a mechanism for checking on the consistency of the input data which is a material requirement if meaningful results are desired. The only disadvantage of this method is that it is computationally more complex. But this disadvantage can be easily eliminated using special software packages prepared to solve AHP applications.

There are also more sophisticated approaches in decision theory such as multi-attribute utility theory (MAUT). In recent years, several articles about the disadvantages of AHP and superiority of MAUT over AHP have been published. Dyer [Ref. 31] points out that AHP suffers rank reversal (an alternative that is chosen as the best over a set X is not chosen when some alternative, perhaps an unimportant one, is excluded from X). He concluded that this problem can be solved by changing ratio scales with interval scales as in MAUT. Perez [Ref. 32] gave the comparison of the two methods and stated:

> One would expect MAUT, since it requires only the construction of an interval scale, to be suitable for a wider range of applications than AHP. However, one would also expect that AHP, since it builds a ratio scale, would be more suitable to some situations in which the subjacent structure had a strong distributive component, particularly those in which the coefficients of the distribution were not strongly affected by changes in the set of available alternatives.

The other shortcoming stated by Dyer [Ref. 31] is the scaling method of AHP. The replies to these criticisms by Saaty [Ref. 33], Harker and Vargas [Ref. 34], and the corresponding counterreplies show that no consensus has been reached. There are also some advantages of AHP over MAUT. Davies [Ref. 35] stated, "AHP is the only decision-making methodology dealing formally with inconsistency of judgments and is, therefore, superior to multi-attribute utility."

Lee and Ahn [Ref. 36] used the AHP method for selecting ground warfare weapons for the Korean Army similar to the TNSG problem. AHP also has been successfully applied to many resource allocation problems similar to the problem of

determining the force structure of TNSG. It is the method selected here for quantifying subjective judgments.

IV. GOAL PROGRAMMING MODELS

There are three basic steps required in formulating a goal program. These steps are: define the decision variables and constants; formulate the constraints (system constraints and goal constraints); and develop the achievement (objective) function. This chapter will describe two main mixed-integer, goal programming (GP) models developed using these three steps.

Model 1 is formulated to determine the number and mix of ship types to purchase or build given a constant budget and weighted AHP preferences for each ship type. In this model, the main goal is to maximize the effectiveness of the force mix when a constant budget is given. Model 2's formulation uses the weapon requirements needed to meet the threat's specific capabilities (defined in the mission areas of air, surface, submarine, and amphibious operations), ship weapon capacities for these requirements, and the weighted AHP preferences for the ship types. In the second model, the main goal is to minimize the cost of providing a fixed force effectiveness. After presenting the details of the models, we describe the motivation in this case for the three steps in model formulation stated above.

A. DETAILED MODEL FORMULATIONS

The goal programming models 1 and 2 are shown in Figures 4.1 and 4.2 below by using the above methodology. In subsequent chapters, model 1 will be referred to as the budget constrained, AHP-preferred (or budget/AHP) model.

Achievement Function

Minimize: $\sum$ (AHP Priority Weight for Ship Type i *
Negative Deviation for Ship Type i)
= Total Deviation

Goal Constraint

Subject to:

X_i - Positive Deviation from Number of Ships of Type i Required
+ Negative Deviation from Number of Ships of Type i Required
= Unattainable Goal

System Constraint

$\sum (X_i * Cost_i) \leq$ Total Budget

X, Positive and Negative Deviations are Integer Variables ≥ 0

Figure 4.1 Model 1 Goal Programming Formulation.

In NPS format, Model 1 is as follows:

Indices :

i: Type of combat ships (CG, DDG, FFG, FSG, PBFG);

w: Priority weights;

Data :

$COST_i$: Cost of each ships of type i in billions of dollars;

$WEIGHTS_{iw}$: AHP weights for each ship type;

NUM_i : Desired number of ships of type i (unattainable);

TOTBUDGET : Total budget available;

Decision Variables :

X_i : Number of ships of type i to purchase;

$DEVNEG_i$: Negative deviation from desired number of ships of type i;

$DEVPOS_i$: Positive deviation from desired number of ships of type i;

DEVIATION : Deviation from objective function;

Model Formulation :

Minimize DEVIATION = $\sum_i DEVNEG_i * \sum_w WEIGHTS_{iw}$ (Objective function)

Subject to $X_i - DEVNEG_i + DEVPOS = NUM_i \quad \forall i$ (Goal constraints)

$\sum_i COST_i * X_i \leq TOTBUDGET$ (System constraints)

$X_i, DEVNEG_i, DEVPOS_i \geq 0$ and integer

Figure 4.2 presents Model 2. In subsequent chapters, it will be referred to as the weapon-constrained model. The AHP priority weight for ship type is summed across the AHP weight for all categories.

Achievement Function

Minimize: $\sum$ (AHP Priority Weight for Ship Type i *
Negative Deviation for Ship Type i)
= Total Deviation

Goal Constraint

Subject to:
X_i - Positive Deviation from Number of Ships of Type i Required
+ Negative Deviation from Number of Ships of Type i Required
= Unattainable Goal

System Constraint

$\sum$ (X_i *capacity for Long Range SSM) $\geq$ Lower Limit of Long Range SSM Required
$\sum$ (X_i *capacity for Long Range SSM) $\leq$ Upper Limit of Long Range SSM Required
$\sum$ (X_i *capacity for Short Range SSM) $\geq$ Lower Limit of Short Range SSM Required
$\sum$ (X_i *capacity for Short Range SSM) $\leq$ Upper Limit of Short Range SSM Required
$\sum$ (X_i *capacity for Long Range SAM) $\geq$ Lower Limit of Long Range SAM Required
$\sum$ (X_i *capacity for Long Range SAM) $\leq$ Upper Limit of Long Range SAM Required
$\sum$ (X_i *capacity for Short Range SAM) $\geq$ Lower Limit of Short Range SAM Required
$\sum$ (X_i *capacity for Short Range SAM) $\leq$ Upper Limit of Short Range SAM Required
$\sum$ (X_i *capacity for Long Range ASM) $\geq$ Lower Limit of Long Range ASM Required
$\sum$ (X_i *capacity for Long Range ASM) $\leq$ Upper Limit of Long Range ASM Required
$\sum$ (X_i *capacity for Short Range ASM) $\geq$ Lower Limit of Short Range ASM Required
$\sum$ (X_i *capacity for Short Range ASM) $\leq$ Upper Limit of Short Range ASM Required
$\sum$ (X_i *capacity for Long Range Guns) $\geq$ Lower Limit of Long Range Guns Required
$\sum$ (X_i *capacity for Long Range Guns) $\leq$ Upper Limit of Long Range Guns Required
$\sum$ (X_i *capacity for Short Range Guns) $\geq$ Lower Limit of Short Range Guns Required
$\sum$ (X_i *capacity for Short Range Guns) $\leq$ Upper Limit of Short Range Guns Required
X, Positive and Negative Deviations are Integer Variables ≥ 0

Figure 4.2 Model 2 Goal Programming Formulation.

In NPS format, Model 2 is as follows:

Indices :

i: Type of combat ships (CG, DDG, FFG, FSG, PBFG);

p: Parameters for each ship type (weapon types which each ship type contains);

w: Priority weights;

Data :

$VALUES_{ip}$: parameter values for each ship type;

$WEIGHTS_{iw}$: AHP weights for each ship type;

NUM_i : Desired number of ships of type i (unattainable);

$LOWREQ_p$: Lower limit (number) of launchers required from weapon type p;

$UPREQ_p$: Upper limit (number) of launchers required from weapon type p;

TOTBUDGET : Total budget available;

Decision Variables :

X_i : Number of ships of type i to purchase;

$DEVNEG_i$: Negative deviation from desired number of ships of type i;

$DEVPOS_i$: Positive deviation from desired number of ships of type i;

DEVIATION : Deviation from objective function;

Model Formulation :

Minimize DEVIATION = $\sum_i DEVNEG_i * \sum_w WEIGHTS_{iw}$ (Objective function)

Subject to $X_i - DEVNEG_i + DEVPOS = NUM_i \quad \forall i$ (Goal constraints)

$$\sum_i VALUES_{ip} * X_i \geq LOWREQ_p \qquad \forall p \qquad \text{(System constraints)}$$

$$\sum_i VALUES_{ip} * X_i L \leq UPREQ_p \qquad \forall p \qquad \text{(System constraints)}$$

$$X_i, DEVNEG_i, DEVPOS_i \geq 0 \quad \text{and integer}$$

B. DECISION VARIABLES AND CONSTANTS

Determination of the decision variables and right hand side is the first step in construction of a goal programming model. The right hand side constants may be either

resource constraints or specified goal levels. Decision variables are controlled by the decision maker and are sometimes referred to as "control variables". Decision variables also must be nonnegative. In both models, the main goal is to find the optimum number and the optimum mix of ship types. In these GP models the decision variables, denoted by X(i), represent the numbers of ships of type i to purchase (i= CG, DDG, FFG, FSG, PBFA). These variables are nonnegative integer variables.

In each goal constraint, there is a negative and positive deviation variable that represents the amounts by which the goals are overachieved (positive deviation) or underachieved (negative deviation). These are also nonnegative integer variables.

In Model 1, constants are the numbers of the ship types desired and the total budget. The total budget is the amount which will be used to structure a new Naval surface force in the next five years. The numbers of ships desired were set as unattainable goals for both models. Model 1 ensures that the total budget is spent in order to reach the goal. Model 2 ensures that all weapon requirements will be met. These weapon requirements are the numbers of weapon launchers for specific missions to meet the threat countries' capabilities.

C. CONSTRAINTS

When analyzing the relationships among decision variables and their relationships to the goals, a set of constraints should be formulated. There are two types of constraints in a goal programming formulation, system constraints and goal constraints. System constraints are absolute constraints, and the decision maker has no control over these constraints. System constraints must be satisfied before an optimal or satisfactory solution can be considered.

Goal constraints are not absolute constraints. These constraints contain positive and negative deviation variables, because goal programming attempts to minimize the deviation from the goal rather than attempt to satisfy the goal completely. The solution could result in overachieving or underachieving the particular goal.

Each goal constraint in Model 1 requires an unattainably high number of each ship type. This will result in a negative deviation from the desired (unattainable) amounts in

order to force a positive deviation which can be minimized in the objective function. The objective function will try to minimize this positive deviation. The deviation is the penalty for underachieving the desired number of ships which is weighted by the ship's AHP preference weight. Model 2 has exactly the same goal constraints as Model 1.

The system constraint in Model 1 is concerned with the total money available (our budget) for the ship types selected. This constraint states that the total cost of the selected ship mix can not exceed our total budget. The total budget is used to structure a new Naval surface force over the next five years. The system constraints in Model 2 are different from the Model 1. These constraints require specific numbers of weapon launchers for each mission (AAW, ASUW, ASW, and Amphibious Warfare, also divided into short and long range). Values for these requirement levels will be determined by the Turkish Naval Surface Group Staff by examining the force structure and weapon capabilities of the threat countries. But in this formulation, dummy values are used as requirement values and budget. Hence, this model is a generic model and can be used by any country by changing these requirements and budgets.

A goal programming model requires that all decision variables be greater than or equal to zero. The negative and positive deviation variables and all other variables used in the goal constraints must be nonnegative.

D. OBJECTIVE FUNCTION

The final step in the model development is to state the objective function. All constraints and goals must be completely identified in the model in order to analyze each goal in terms of whether over or underachievement of the goal is satisfactory or not. If overachievement is satisfactory (as in Model 1 and 2), the positive deviation variables can be eliminated from our objective function. Likewise, if underachievement is acceptable, the negative deviation variables can be eliminated from objective function. If exact achievement is required, then both negative and positive deviation variables will be in the objective function. In both models the objective function minimizes the sum of negative deviations from the number of ships of varying types weighted by the AHP priority vector.

E. VARIATIONS ON BASIC MODELS

A variation of Model 1 can be formulated to represent a least cost, war situation (minimum deviation from the budget). The scenario on which weapon requirements are based is a conventional war situation with a specific country. This model formulation can be used to determine the least cost mix of ship types to meet the total weapon requirements in a war situation. Values for weapon requirement levels will also be determined as in Model 2. This model will not use any AHP weights, and it is simply a linear programming formulation.

The objective functions of the models used are linear (first-order) functions. But linear utility functions can give irrelevant results when there are strong interactions between factors (this interdependency problem is solved in our models by using structural constraints). Quadratic (second-order) objective function models can also be used to solve this interaction problem. The AHP models are modified using a quadratic objective function and are evaluated in our determination of the force mix of TNSG.

V. SOFTWARE SUPPORT AND MODEL RESULTS

The primary objective of this study is to develop a model and to use this model to determine the optimum force structure of TNSG. This chapter is intended to present the computer programs used to solve the model and the model results. Specifically, this entails a discussion of the data collection and software development process, the numerical output acquired from the various models' iterations and the outcomes produced.

A. DATA COLLECTION AND SOFTWARE DEVELOPMENT

The process of collecting data and designing software to support the GP model formulation is an essential step in determining force structure of the TNSG. As discussed earlier, the GP model requires that the objective function weights be determined by the AHP. The AHP survey was intended to determine a relative measure of capability of the ships in several warfare situations and factors. The survey was based on recommendations by Saaty [Ref. 29] for eliciting preferences of paired comparisons. The forms were distributed to the 20 Turkish Navy Officers attending the Naval Postgraduate School. Fifteen of the surveys were completed. It is assumed that those officers who did complete the survey are qualified to make these judgments. A copy of the actual surveys can be found in Appendix A.

Computer software was used to perform the necessary AHP calculations and determine the objective function coefficients of the GP model. The Microsoft Excel V.7.0 spreadsheet and S-Plus V.3.3 Release 1 statistical package by Mathsoft [Ref. 37] were chosen to perform AHP calculations. Excel especially, is easy to use in many statistical calculations. The eigenvalues were calculated by Excel. The eigenvectors and consistency ratios were calculated by S-Plus.

The Generalized Algebraic Modeling System (GAMS 2.25) [Ref. 38] was selected to solve goal programming models. GAMS is designed to ease the construction and solution of large and complex mathematical programming models. The programs are formulated as mixed integer problems (MIP) and require the use of the XA subsolver of

GAMS that is the implementation of the Sunset Technology XA Callable Library. XA contains high performance solvers for LP and MIP. The XA solver first solves the problem as a linear program. Then it uses the Pivot and Complement heuristic to find an initial integer feasible solution. Finally XA, uses a branch and bound algorithm to determine whether there is a better solution and to verify optimality [Ref. 39].

B. MODEL RESULTS

The output from the AHP procedure and the GP models will be surveyed in this section. This evaluation will include a comparison of different GP models and selection of an optimal model.

1. AHP Results

The AHP surveys were collected and the pairwise comparison matrices were obtained by taking the geometric mean as recommended by Saaty [Ref. 29] . These matrices were put on Excel and S-Plus to generate the final AHP results. The equal preference weights are given for all ship characteristics in computation of the AHP results. The AHP comparison matrices for each ship characteristic (cost, speed, warfare capabilities, surveillance capability, and fuel consumption) and AHP results (AHP λ values, consistency indices, consistency ratios and the eigenvectors) are presented in Appendix B. Table 5.1 shows the composite matrix of the AHP values.

Ship	Cost	Speed	AAW Cap	ASW Cap.	ASUW Cap.	AMPW Cap.	CIWS Cap.	Fuel Con.	Surv. Cap.
CG	.049	.132	.375	.238	.319	.388	.287	.089	.356
DDG	.091	.152	.272	.331	.221	.270	.263	.169	.257
FFG	.181	.169	227	.297	.220	.210	.255	.200	.232
FSG	.289	.220	.087	.099	.135	.091	.148	.226	.103
PBFA	.390	.327	.039	.035	.106	.041	.047	.316	.051

Table 5.1 Composite Matrix of AHP Results.

Consistency ratios for all comparison matrices were well under the recommended value of 10 percent. The highest consistency ratio was only 3.5 percent. It is critical to the

GP model that the standard for consistency not be violated, since inconsistency will adversely affect the accuracy and credibility of the model. The CG was preferred in five of nine categories.

2. Goal Programming Results

As indicated earlier, dummy data values are used in this study to make these models generic and usable for other countries. Appendices C through G show the GAMS formulation and summarized output.

Appendix C contains the formulation and results of Model 1. It is a budget-constrained, AHP-preferred mix goal to determine the mix and the number of ship types. The resulting mix consisted of 20 PBFA and 1 FSG; no other types of ships are included. All of the budget is used; there was no remaining budget. When the AHP preferences are compared with the ship costs, the PBFA is the most preferred ship type in the model. A variation of Model 1 was formulated to represent a war situation, least cost (minimum deviation from the budget) goal. The model formulation is used to determine the least cost mix of ship types to meet the total weapon requirements in a war situation. This model is shown in Appendix D, and the ships selected were 2 DDGs, 6 FFGs, 4 FSGs, and 1 PBFA. The multi-mission capability and cost (when compared with the other multi-mission ship types) of the FFGs are the main reasons for the selection of this type.

Model 2 (Appendix E) was formulated using a weapon constrained, AHP-preferred mix goal. The AHP preferences and weapon requirements drove the selected mix of ships. This mix was 8 FFGs, 2 FSGs, and 3 PBFAs. The addition of AHP preferences increased the number of FFGs and PBFAs.

Table 5.2 summarizes the model results. As can be seen, the objective value of the budget/AHP model is the smaller than the AHP-preferred model (the same goals and objective function). The appearance of all PBFAs in the budget/AHP solution is explained by the high ratio of AHP weight values to the cost for this ship type. This results in the minimum deviation value in this model. But PBFAs are not multi-mission surface combatants, and the mix with only this type of ship seems inappropriate for this problem.

	Weapon-Constrained Models		
Ship Type	Budget AHP	Minimum Cost	AHP Preferred
CG	--	--	--
DDG	--	2	--
FFG	--	6	8
FSG	1	4	2
PBFA	20	1	3
Objective Value	151.56 (ships)	1.13 ($\$10^9$)	157.22 (ships)

Table 5.2 Gams GP Model Results Summary.

The evaluation of the weapon-constrained models can be done by comparing the ship types on the basis of cost and benefit. Table 5.3 summarizes the results of this comparison. The sum of each ship type's eigenvalues are normalized to one

Ship Type	Marginal Cost	Marginal Benefit	Minimum Cost	AHP Preferred
CG	0.9	0.258	--	--
DDG	0.6	0.232	2	--
FFG	0.3	0.218	6	8
FSG	0.2	0.152	4	2
PBFA	0.13	0.140	1	3
Total Mix	----	----	13	13
Total Mix Cost ($\$10^9$)	----	----	3.93	3.19
Total Mix Benefit	----	----	2.52	2.47
Benefit-Cost Ratio	----	----	0.64	0.77

Table 5.3 Weapon-Constrained Models Comparison.

and used as a measure of marginal benefit. It can be seen that the AHP-Preferred mix, while costing 0.8 billion dollars less, has approximately equal total benefit with the minimum cost model and has a larger benefit-cost ratio. The minimum cost model is the only formulation not using AHP weights, and it has the least benefit-cost ratio. Our

evaluation of the models shows that the AHP-Preferred model gives the most appropriate mix for the TNSG. These results provide an indication of the overall superiority of FFG-dominated ship mixes. The cost-benefit ratio calculations were made using results of the AHP-Preferred and weapon constrained model. The use of the AHP cost weights in the cost-benefit calculations may be viewed as away of accounting for cost when firm cost estimates are not available. Future research on this issue is meredit.

The budget-constrained AHP model can be modified using a quadratic objective function with the same hypothetical AHP values. The GAMS formulation of this new model and its output are shown in Appendix F. Unlike Model 1 (budget-constarined, AHP-preferred), this quadratic programming (QP) model was not dominated by PBFAs (Model 1 mix contains 20 PBFA and 1 FSG, QP model mix contains 4 FFG, 4 FSG, and 5 PBFA). QP results are more similar to weapon-constrained model and they can be used in future studies.

VI. SENSITIVITY ANALYSIS

In previous chapters, three different models were developed based on several data values and objective function weights obtained from the surveys. It is obvious that responses to these surveys may not be perfect; the data used may be subject to error, and ship capabilities can change with time. The purpose of this chapter is to analyze the impacts of changes in the goal programming model. There will be sensitivity analyses of the following changes: total budget, weapon requirements, and objective function coefficients (weights).

A. CHANGES IN BUDGET

Model 1 (budget-constrained and AHP-preferred) is designed to find a surface force mix with a budget constraint and AHP weights. The possible budget for force structuring is a five year plan, and it can change over time due to several effects (political, economic, etc.). Possible budget levels were run in the model and resulted in different solutions. The solutions showed that the number of ships increase (decrease) with an increase (decrease) in the budget, but the ship types that are selected by the model did not change.

The least-cost model was also run at several different budget levels. Every budget level gave the same solution to the model even with a changing objective function. The objective function value, the weighted deviation, increases (decreases) with decreases (increases) in the budget. It showed that the least cost model is not sensitive to changes in the budget.

B. CHANGES IN WEAPON REQUIREMENTS

Weapon requirement constraints force specific numbers of weapon launchers for each mission (AAW, ASUW, ASW, and Amphibious Warfare, also divided into short and long range).Values for these requirement levels will be determined by the Turkish Naval Surface Group Staff after examining the force structure and weapon capabilities of the threat countries. These requirements can change with a force structure change of the

threat countries and/or addition of new threats (for example, the threat country can decide to reduce or increase its force size).

The minimum weapon requirements must first be satisfied for all warfare areas. We will try to determine the effects of changes in the weapon requirements. Every constraint value is changed using a loop in the GAMS model. The following results were found.

The analysis of the requirements for long and short range surface to surface missiles (NLSMUP: the upper bound for the number of long range surface to surface missiles, NLSMLO: the lower bound for the number of long range surface to surface missiles, NSSMUP: the upper bound for the number of short range surface to surface missiles, and NSSMLO: the lower bound for the number of short range surface to surface missiles) gave the results that are shown in Table 6.1. Table 6.1 shows that changes in the lower bounds of these constraints do not have any effects on the solution of the problem. These constraints are likely to be redundant. However, changes in the upper bounds have significant effects on the determination of the force mix. Decreases in the upper bounds

Constraint	Right Hand Side	Obj. Func. Value	Force Mix
NLSMUP	55-52	157.22	8 FFG, 2 FSG, 3 PBFA
	51-48	160.48	1 DDG, 6 FFG, 3 FSG, 1 PBFA
	47	NO FEASIBLE INTEGER SOLUTION	
NLSMLO	30-52	157.22	8 FFG, 2 FSG, 3 PBFA
	53	NO FEASIBLE INTEGER SOLUTION	
NSSMUP	54-52	157.22	8 FFG, 2 FSG, 3 PBFA
	51-44	158.23	1 DDG, 7 FFG, 2 FSG, 2 PBFA
	43-36	159.85	2 DDG, 6 FFG, 2 FSG, 1 PBFA
	35-32	161.80	2 DDG, 5 FFG, 3 FSG
	31	NO FEASIBLE INTEGER SOLUTION	
NSSMLO	30-52	157.22	8 FFG, 2 FSG, 3 PBFA
	53	NO FEASIBLE INTEGER SOLUTION	

Table 6.1 Sensitivity Analysis of Long and Short Range Surface to Surface Missile Requirements.

result in lower numbers of ships in the force mix which results in bigger objective function values.

The analysis of the requirements for long and short range surface to air missiles (constraints: NLAMUP, NLAMLO, NSAMUP, and NSAMLO) gave the results that are shown in Table 6.2. Table 6.2 shows that a change in the upper bound of long range surface to air missile (SAM) requirements does not have any effect on the solution of the problem. Again, this constraint is likely to be redundant. The changes in the lower bound of long range SAM and upper bound of short range SAM requirements have significant effects on the determination of the force mix. The increase in the lower bound of short range SAM does not have any effect in the force mix, but at some point the problem becomes infeasible.

Constraint	Right Hand Side	Obj. Func. Value	Force Mix
NLAMUP	55-32	157.22	8 FFG, 2 FSG, 3 PBFA
	31	NO FEASIBLE INTEGER SOLUTION	
NLAMLO	30-32	157.22	8 FFG, 2 FSG, 3 PBFA
	33-36	158.58	1 DDG, 7 FFG, 1 FSG, 3 PBFA
	37-40	159.85	2 DDG, 6 FFG, 2 FSG, 1 PBFA
	41	NO FEASIBLE INTEGER SOLUTION	
NSAMUP	55-40	157.22	8 FFG, 2 FSG, 3 PBFA
	39-36	158.37	1 DDG, 7 FFG, 2 FSG, 2 PBFA
	35-32	158.58	1 DDG, 7 FFG, 1 FSG, 3 PBFA
	31	NO FEASIBLE INTEGER SOLUTION	
NSAMLO	31-40	157.22	8 FFG, 2 FSG, 3 PBFA
	41	NO FEASIBLE INTEGER SOLUTION	

Table 6.2 Sensitivity Analysis of Long and Short Range Surface to Air Missile Requirements.

Table 6.3 shows the analysis of the changes in the long and short range antisubmarine weapon requirements (constraints: NLASMUP, NLASMLO, NSASMUP, and NSASMLO). A change in the upper bound of long range antisubmarine weapon

requirements does not have any effect on the solution of the problem; this constraint is also likely to be redundant. The changes in the other antisubmarine weapon requirements have significant effects on determination of the force mix. The lower bound of short range antisubmarine weapons results in very sensitive mix changes.

Constraint	Right Hand Side	Obj. Func. Value	Force Mix
NLASMUP	41-24	157.22	8 FFG, 2 FSG, 3 PBFA
	23	NO FEASIBLE INTEGER SOLUTION	
NLASMLO	24	157.22	8 FFG, 2 FSG, 3 PBFA
	25-27	158.54	1 DDG, 7 FFG, 2 FSG, 2 PBFA
	28-29	159.58	1 DDG, 7 FFG, 1 FSG, 3 PBFA
	30-34	158.54	1 DDG, 7 FFG, 2 FSG, 2 PBFA
	35	NO FEASIBLE INTEGER SOLUTION	
NSASMUP	41-36	157.22	8 FFG, 2 FSG, 3 PBFA
	35-30	158.58	1 DDG, 7 FFG, 1 FSG, 3 PBFA
	29	NO FEASIBLE INTEGER SOLUTION	
NSASMLO	24-36	157.22	8 FFG, 2 FSG, 3 PBFA
	37-39	159.13	1 DDG, 6 FFG, 3 FSG, 2 PBFA
	40	NO FEASIBLE INTEGER SOLUTION	

Table 6.3 Sensitivity Analysis of Long and Short Range Anti Submarine Weapons Requirements.

Table 6.4 shows the analysis of the changes in the requirements of long and short range guns (constraints: NLGUNSUP, NLGUNSLO, NSGUNSUP, and NSGUNSLO). Table 6.4 shows that changes in the upper bounds of long and short range guns have no effects on the solution of the problem. These constraints are also likely to be redundant. The changes in the lower bounds of long and short range guns have significant effects on the force mix. These requirements are likely to be dominant constraints that affect the number of other weapons.

Constraint	Right Hand Side	Obj. Func. Value	Force Mix
NLGUNSUP	15-10	157.22	8 FFG, 2 FSG, 3 PBFA
NLGUNSLO	5	157.26	8 FFG, 1 FSG, 4 PBFA
	6	157.22	8 FFG, 2 FSG, 3 PBFA
	7	157.26	8 FFG, 2 FSG, 4 PBFA
	8-10	157.22	8 FFG, 2 FSG, 3 PBFA
	11	158.54	1 DDG, 7 FFG, 2 FSG, 2 PBFA
	12	159.85	2 DDG, 6 FFG, 2 FSG, 1 PBFA
	13	NO FEASIBLE INTEGER SOLUTION	
NSGUNSUP	15-10	157.22	8 FFG, 2 FSG, 3 PBFA
NSGUNSLO	5-6	156.4	10 FFG, 3 PBFA
	7-8	156.58	9 FFG, 2 FSG, 2 PBFA
	9-10	157.22	8 FFG, 2 FSG, 3 PBFA
	11-12	159.13	1 DDG, 6 FFG, 3 FSG, 2 PBFA
	13	NO FEASIBLE INTEGER SOLUTION	

Table 6.4 Sensitivity Analysis of Long and Short Range Guns Requirements.

As a summary, the sensitivity analysis of weapon requirements show that the changes in the lower bounds of long and short range guns give the most sensitive results. Decreases in the upper bound of the requirements decreases the number of the ships in the force mix. The increases in the lower bound of the long range guns forces the model to include the ship types CG and DDG (the ship types have more long range guns), and this increases the number of long range SSMs (because CG and DDG have more long range SSM). This makes the changes in the upper bound of the long range SSM more sensitive.

C. CHANGES IN OBJECTIVE FUNCTION COEFFICIENTS (WEIGHTS)

Survey results were converted, using the AHP, into objective function coefficients. The survey is based on subjective judgments that can possibly vary for several reasons. In this section an analysis of the impact of the changes in the subjective evaluations will be conducted.

First, a sensitivity analysis can be done to determine the impact of a mistake in completing a survey or transcribing data from a survey. This analysis is illustrated in Table

6.5, a table that displays an actual matrix taken from the AHP output in Appendix B and a flawed matrix that contains an input error. The only change that was made in the flawed matrix is that the upper right-hand number was changed from 0.17 to 1.0, representative of a common input error. The consistency ratio (CR) computation shows that this small mistake increases the CR to 0.168. Since a CR this high (bigger than 0.1) is unacceptable, the matrix data input would have to be examined, and the error would be corrected. This example shows the sensitivity of the GP model in responding to minor lapses.

Accurate Matrix						Flawed Matrix				
1.00	0.50	0.25	0.20	0.17		1.00	0.50	0.25	0.20	1.00
2.00	1.00	0.50	0.25	0.20		2.00	1.00	0.50	0.25	0.20
4.00	2.00	1.00	0.50	0.33		4.00	2.00	1.00	0.50	0.33
5.00	4.00	2.00	1.00	0.51		5.00	4.00	2.00	1.00	0.51
5.95	5.00	2.99	1.95	1.00		1.00	5.00	2.99	1.95	1.00
Consistency Ratio: 0.024						Consistency Ratio: 0.168				

Table 6.5 Comparison of Accurate Matrix with Flawed Matrix.

The GP model might also be subjected to a change of opinion. A respondent to a survey could decide that a ship type was judged inappropriately. This can be shown using an example of such a situation. Consider the comparison matrix of ship types according to cost as shown in Table 6.6.

	CG	DDG	FFG	FSG	PBFA
CG	1.00	0.50	0.25	0.20	0.17
DDG	2.00	1.00	0.50	0.25	0.20
FFG	4.00	2.00	1.00	0.50	0.33
FSG	5.00	4.00	2.00	1.00	0.51
PBFA	5.95	5.00	2.99	1.95	1.00
Consistency Ratio: 0.024					
Eigenvector :	.049	.091	.181	.289	.390

Table 6.6 Original Comparison Matrix for Cost.

Suppose the surface group staff officers receive information implying that new DDGs will have both gas turbine and diesel engines that will increase their cost. The officers might then change the original comparison matrix to one shown in Table 6.7.

	CG	DDG	FFG	FSG	PBFA
CG	1.00	0.75	0.25	0.20	0.17
DDG	1.33	1.00	0.50	0.23	0.19
FFG	4.00	2.50	1.00	0.50	0.33
FSG	5.00	4.44	2.00	1.00	0.51
PBFA	5.95	5.56	2.99	1.95	1.00
Consistency Ratio: 0.012					
Eigenvector :	.054	.071	.188	.293	.394

Table 6.7 Comparison Matrix for Cost with Minor Changes.

The only changes were made in the second row and second column of the new matrix, values corresponding to the DDG. The numbers in the original matrix were larger, signifying the cost of DDGs is not too great; the lower numbers in the new matrix denote that the cost of DDGs has increased significantly. This is also reflected by the coefficient eigenvector. The largest coefficient change occurred in the DDG value, which dropped from 0.091 to 0.071, but the other coefficient weights increased only slightly.

The example above demonstrates the sensitivity of the models to major subjective changes in the responses to the AHP surveys. The new coefficient weights were used in the AHP-preferred models, and the results show that big changes in subjective judgments make big differences in the surface group force mix. These changes have even greater effects in the quadratic GP models.

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The determination of force structure of a naval surface group is essentially a project selection problem. There are additional factors, aside from cost and weapon requirements, which must be included when considering the mix and size of the naval surface group. These factors should include not only the ship attributes such as speed, cost, warfare capabilities, and fuel consumption, but with respect to these attributes, the relative advantage held by each ship in accomplishing the mission. A survey is a method to obtain subjective judgments rating ships with respect to the above factors. The Analytic Hierarchy Process appears to be a useful tool for converting these subjective judgments into numerical preference weights.

Goal programming formulations using AHP-weighted deviation variables can produce a consistent, and in some sense, the best solution for determining force size and mix. This was demonstrated by the smallest goal deviation and the highest benefit-cost ratio achieved by the ship mix selected by the AHP-Preferred model formulation.

Although CGs and DDGs are the most capable ships, the mix does not contain these ship types because of their high cost. FFGs are the most preferable ship types, because they have multi-mission capability and low cost when compared with other multi-mission ship types (CG and DDG). FSGs and PBFAs are preferable because of their low cost, but their warfare capabilities limit their numbers in the force mix.

B. RECOMMENDATIONS FOR FUTURE STUDY

Additional model formulations should be constructed using other surface warfare requirements, such as a helicopter landing capability and operational capability in open sea versus shoreline (i.e., seas with lots of islands like the Aegean Sea). This model formulation can be enlarged with the addition of the other warfare types, such as mine warfare.

The AHP survey should be expanded to include other factors which are thought to contribute to naval surface group mission effectiveness (in several warfare types). Some factors can be divided into subgroups (ASUW capability in open sea or shoreline). It might also prove useful to ascertain the relative importance of mission effectiveness parameters among themselves (e.g., the advantage of AAW warfare capability over fuel consumption). The mixed integer GP models can be converted to quadratic mixed GP models using a different applications of the AHP results. Additionally, the population of survey judges should be increased.

Future studies can include more thorough analysis of AHP versus MAUT. Also, new methods can be obtained by combining the attractive features of both techniques (e.g., multiple judgments and calculation of inconsistency in AHP and the interdependencies in MAUT).

The participation of the ship types in a task force group is another future issue in our study. A nominal task force must be a task-organized group of combatants and includes the interdependencies between ship types that are included in the force. The task group formation must provide all kind of warfare capability in the protection of the all constituent variables.

APPENDIX A. SURVEY

SHIP CHARACTERISTICS SURVEY

This survey was designed to obtain pairwise comparison data of the ships in the naval surface group. The ships are compared on the basis of nine different characteristics. You have been briefed on the purpose of this survey and its theoretical foundations.

The following pairwise comparison scale will be used to compare each type of ship against the others with respect to the characteristics:

1 - equal/same
2
3 - moderate
4
5 - strong
6
7 - very strong
8
9 - extremely strong

The values 2, 4, 6, and 8 are intermediate values.

Please ensure that you circle either advantage or disadvantage when making each comparison.

Example: The CG gives a __3__advantage/disadvantage over the FFG.

Questions may be directed to LTJG Erol Unal, 642-9860.

Comparison 1: Costs. Given the following costs (billions) for each ship type: CG -

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

Comparison 2: Speed. The speed of a ship is a factor that increases the operational capability of the ship in different warfare situations. In addition, speed is a factor in the survivability of the ship.

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

Comparison 3,4,5,6,7: Warfare Capabilities. This shows the capability of the ships in different warfare areas. The warfare capability of the ship changes according to the ship's weapons in that warfare area.

With respect to AAW capability: (AAW weapons and countermeasures)

The CG gives a: _________ advantage/disadvantage over DDG.

_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA

With respect to ASUW capability: (ASUW weapons and countermeasures)

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

With respect to ASW capability: (ASW weapons and countermeasures)

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

With respect to AMPW capability: (Navy gun fire support capability)

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

With respect to CIWS capability:

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA.

Comparison 8: Surveillance capability. (Radars, ESM, and ECM capability)

With respect to Surveillance capability:

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.

_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA

Comparison 9: Fuel consumption.

With respect to fuel consumption:

The CG gives a: _________ advantage/disadvantage over DDG.
_________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The DDG gives a: _________ advantage/disadvantage over FFG.
_________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FFG gives a: _________ advantage/disadvantage over FSG.
_________ advantage/disadvantage over PBFA.

The FSG gives a: _________ advantage/disadvantage over PBFA

APPENDIX B. AHP RESULTS FOR SHIP CHARACTERISTICS

COMPARISON MATRIX FOR COSTS

	CG	DDG	FFG	FSG	PBFA
CG	1.000	0.500	0.250	0.200	0.168
DDG	2.000	1.000	0.500	0.250	0.200
FFG	4.000	2.000	1.000	0.500	0.334
FSG	5.000	4.000	2.000	1.000	0.512
PBFA	5.952	5.000	2.994	1.953	1.000

LAMBDA : 4.892
CONSISTENCY INDEX : 0.027
CONSISTENCY RATIO : 0.024

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.049	0.091	0.181	0.289	0.390

COMPARISON MATRIX FOR AAW CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	1.567	1.925	5.340	8.240
DDG	0.638	1.000	1.500	4.000	6.000
FFG	0.519	0.667	1.000	3.750	5.000
FSG	0.187	0.250	0.267	1.000	2.500
PBFA	0.121	0.167	0.200	0.400	1.000

LAMBDA : 4.942
CONSISTENCY INDEX : 0.0145
CONSISTENCY RATIO : 0.013

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.375	0.272	0.227	0.087	0.039

COMPARISON MATRIX FOR ASUW CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	1.380	1.420	2.500	3.000
DDG	0.725	1.000	1.112	1.500	2.100
FFG	0.704	0.899	1.000	1.600	2.200
FSG	0.400	0.667	0.625	1.000	1.230
PBFA	0.333	0.476	0.455	0.813	1.000

LAMBDA : 5.004
CONSISTENCY INDEX : 0.001
CONSISTENCY RATIO : 0.00089

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.319	0.221	0.220	0.135	0.106

COMPARISON MATRIX FOR ASW CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	0.735	0.806	3.760	5.600
DDG	1.361	1.000	1.142	4.850	8.205
FFG	1.241	0.876	1.000	4.560	7.145
FSG	0.266	0.206	0.219	1.000	3.250
PBFA	0.179	0.122	0.140	0.308	1.000

LAMBDA : 5.063
CONSISTENCY INDEX : 0.0158
CONSISTENCY RATIO : 0.0140

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.238	0.331	0.297	0.099	0.035

COMPARISON MATRIX FOR AMPW CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	2.020	2.940	5.210	7.100
DDG	0.495	1.000	1.540	3.750	5.950
FFG	0.340	0.649	1.000	3.120	4.780
FSG	0.192	0.267	0.321	1.000	2.510
PBFA	0.141	0.168	0.209	0.398	1.000

LAMBDA : 5.082
CONSISTENCY INDEX : 0.0205
CONSISTENCY RATIO : 0.0183

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.388	0.270	0.210	0.091	0.041

COMPARISON MATRIX FOR CIWS CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	1.123	1.135	2.105	5.890
DDG	0.890	1.000	1.005	1.895	5.552
FFG	0.881	0.995	1.000	1.723	5.425
FSG	0.475	0.528	0.580	1.000	3.230
PBFA	0.170	0.180	0.184	0.310	1.000

LAMBDA : 5.001
CONSISTENCY INDEX : 0.00025
CONSISTENCY RATIO : 0.00022

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.287	0.263	0.255	0.148	0.047

COMPARISON SURVEILLANCE CAPABILITY

	CG	DDG	FFG	FSG	PBFA
CG	1.000	1.967	2.236	3.756	5.456
DDG	0.508	1.000	1.112	3.128	4.654
FFG	0.447	0.899	1.000	2.792	4.234
FSG	0.266	0.320	0.358	1.000	2.234
PBFA	0.183	0.215	0.236	0.448	1.000

LAMBDA : 5.062
CONSISTENCY INDEX : 0.0155
CONSISTENCY RATIO : 0.0139

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.356	0.257	0.232	0.103	0.051

COMPARISON MATRIX FOR SPEED

	CG	DDG	FFG	FSG	PBFA
CG	1.000	0.890	0.875	0.542	0.363
DDG	1.124	1.000	0.902	0.757	0.436
FFG	1.143	1.109	1.000	0.867	0.567
FSG	1.845	1.321	1.153	1.000	0.789
PBFA	2.755	2.294	1.764	1.267	1.000

LAMBDA : 5.018
CONSISTENCY INDEX : 0.0045
CONSISTENCY RATIO : 0.0040

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.132	0.152	0.169	0.220	0.327

COMPARISON MATRIX FOR FUEL CONSUMPTION

	CG	DDG	FFG	FSG	PBFA
CG	1.000	0.512	0.418	0.398	0.332
DDG	1.953	1.000	0.884	0.689	0.496
FFG	2.392	1.131	1.000	0.857	0.568
FSG	2.513	1.451	1.167	1.000	0.612
PBFA	3.012	2.016	1.761	1.634	1.000

LAMBDA : 5.014
CONSISTENCY INDEX : 0.0035
CONSISTENCY RATIO : 0.0031

	CG	DDG	FFG	FSG	PBFA
WEIGHT EIGENVECTOR :	0.089	0.169	0.200	0.226	0.316

APPENDIX C. GAMS GOAL PROGRAMMING FORMULATION AND OUTPUT FOR BUDGET-CONSTRAINED AND AHP PREFERRED MIX MODEL

A. GAMS FORMULATION

```
SETS
   I   type of combat ships / CG        cruisers
                            DDG        guided missile destroyers
                            FFG        guided missile frigates
                            FSG        guided missile corvettes
                            PBFA       guided missile fast patrol boats   /

   P   parameters for each ship type
       / C   cost of ship type I in billions of dollars /
   W   priority weights
       / CW           cost weights
         SW           speed weights
         AAWW        aaw  capability weights
         ASWW        asw  capability weights
         ASUWW       asuw capability weights
         AMPW        amphibious warfare capability weights
         CIWS         ciws capability weights
         FW           fuel consumption weights
         SCW          surveillance capability weights  /;

 TABLE
      VALUES (I,P) parameter values for each ship type

                 C   LSM   SSM   LAM   SAM   LASM   LSSM   LGUNS   GUNS
      CG        .9    16     0     8     0      8      6       2      2
      DDG       .6     8     0     8     0      8      3       2      2
      FFG       .3     4     4     4     4      0      6       1      0
      FSG      .20     4     4     0     4      0      6       1      2
      PBFA     .13     4     4     0     0      0      0       0      2
;
TABLE
      WEIGHTS (I,W) AHP weights  for each ship type

              CW     SW    AAW   ASWW   ASUWW   AMPW   CIWS    FW    SCW

      CG     .049   .132   .375   .238    .319    .388   .287   .089   .356
      DDG    .091   .152   .272   .331    .221    .270   .263   .169   .257
      FFG    .181   .169   .227   .297    .220    .210   .255   .200   .232
      FSG    .289   .220   .087   .099    .135    .091   .148   .226   .103
```

```
      PBFA  .390   .327   .039      .035        .106        .041      .047   .316   .051
;
PARAMETER
     NUM (I)      desired number of ship type I (unattainable)
          / CG      20
            DDG    20
            FFG    20
            FSG    20
            PBFA   20                   /;
SCALARS
     TOTBUDGET        / 2.8  /;
INTEGER VARIABLES
     X (I)                  number of ships I to purchase
     DEVNEG (I)        negative deviation from desired number of ships of  type I
     DEVPOS (I)         positive deviation from desired number of ships of  type I   ;
POSITIVE VARIABLE
     DEVIATION       deviation from objective function ;
EQUATIONS
     BUDGET            stay within budget
     NSHIPS(I)          desired number of ship type I (unattainable)
     OBJECFUN         objective function;

     BUDGET..          SUM(I, X(I) * VALUES(I,'C'))  =L= TOTBUDGET   ;
     NSHIPS(I)..       X(I) - DEVPOS(I) + DEVNEG(I) =E= NUM(I) ;
     OBJECFUN..         DEVNEG( 'CG' ) * SUM(W,WEIGHTS('CG',W))
                      + DEVNEG( 'DDG' ) * SUM(W,WEIGHTS('DDG',W))
                      + DEVNEG( 'FFG' ) * SUM(W,WEIGHTS('FFG',W))
                      + DEVNEG( 'FSG' ) * SUM(W,WEIGHTS('FSG',W))
                      + DEVNEG( 'PBFA' ) * SUM(W,WEIGHTS('PBFA',W))
                      =E= DEVIATION ;
MODEL GP /ALL/;
SOLVE GP USING MIP MINIMIZING DEVIATION;
```

B. GAMS MIXED INTEGER PROBLEM OUTPUT

```
                    S O L V E     S U M M A R Y

   MODEL   GP              OBJECTIVE  DEVIATION
   TYPE    MIP             DIRECTION  MINIMIZE
   SOLVER  XA               FROM LINE  78
```

```
**** SOLVER STATUS        1 NORMAL COMPLETION
**** MODEL STATUS         1 OPTIMAL
**** OBJECTIVE VALUE            151.5620

 RESOURCE USAGE, LIMIT          0.000    50000.000
 ITERATION COUNT, LIMIT         5            20000

No better solution than :   151.56200
                          Absolute                 Relative
Actual distance           0.00000                  0.00000
Tolerances   (OPTCA)      0.00000   (OPTCR)        0.00000

                          LOWER     LEVEL     UPPER    MARGINAL

---- EQU BUDGET           -INF      2.800     2.800      -6.990

                          LOWER     LEVEL     UPPER    MARGINAL
---- EQU OBJECFUN            .         .         .       -1.000

---- VAR X          number of ships I to purchase

             LOWER     LEVEL     UPPER    MARGINAL

CG             .         .      100.000     4.058
DDG            .         .      100.000     2.168
FFG            .         .      100.000     0.106
FSG            .       1.000    100.000       .
PBFA           .      20.000    100.000       .

---- VAR DEVNEG      negative deviation from desired number of ships of  type I

           LOWER     LEVEL      UPPER    MARGINAL

CG           .       20.000    100.000       .
DDG          .       20.000    100.000       .
FFG          .       20.000    100.000       .
FSG          .       19.000    100.000       .
PBFA         .         .       100.000     0.043

---- VAR DEVPOS      positive deviation from desired number of ships of  type I

            LOWER     LEVEL     UPPER    MARGINAL
```

CG	.	.	100.000	2.233
DDG	.	.	100.000	2.026
FFG	.	.	100.000	1.991
FSG	.	.	100.000	1.398
PBFA	.	.	100.000	0.909

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR DEVIATION	-INF	151.562	+INF	.

APPENDIX D. GAMS GOAL PROGRAMMING FORMULATION AND OUTPUT FOR THREAT-CONSTRAINED (LEAST COST) MODEL

A. GAMS FORMULATION

```
SETS
    I   type of combat ships / CG        cruisers
                               DDG       guided missile destroyers
                               FFG       guided missile frigates
                               FSG       guided missile corvettes
                               PBFA      guided missile fast patrol boats   /

    P   parameters for each ship type
        / C        cost of ship type I in billions of dollars /
          LSM      number of the long  range surface to surface missiles
          SSM      number of the short  range surface to surface missiles
          LAM      number of the long range surface to air missiles
          SAM      number of the short range surface to air missiles
          LASM     number of the long range anti submarine rockets or torpedoes
          SASM     number of the short range anti submarine rockets or torpedoes
          LGUNS    number of the long range  guns
          SGUNS    number of the short range guns   /;

TABLE
VALUES (I,P) parameter values for each ship type

               C    LSM   SSM    LAM    SAM   LASM   LSSM   LGUNS   GUNS
     CG       .9    16     0      8      0     8      6       2       2
     DDG      .6     8     0      8      0     8      3       2       2
     FFG      .3     4     4      4      4     0      6       1       0
     FSG      .2     4     4      0      4     0      6       1       2
     PBFA    .13     4     4      0      0     0      0       0       2

;

SCALARS
     TOTBUDGET      / 2.8  /;

POSITIVE VARIABLES
     DEVIATION      deviation from objective function
     DEVNEGB        negative deviation from budget
     DEVPOSB        positive deviation from budget   ;

INTEGER VARIABLE
     X(I)           number of ships I to purchase   ;

EQUATIONS
      NUMLSM        number of lsm launchers needed to meet threat's surface power
```

```
NUMSSM        number of ssm launchers needed to meet threat's surface power
NUMLAM        number of lam launchers needed to meet threat's air power
NUMSAM        number of sam launchers needed to meet threat's air power
NUMLASM       number of lasm launchers needed to meet threat's submarines
NUMSASM       number of sasm launchers needed to meet threat's submarines
NUMLGUNS      number of lguns needed to meet threat's power
NUMSGUNS      number of sguns needed to meet threat's power
BUDGET        stay within budget
OBJFUN        objective function;
NUMLSM..      SUM( I, X(I) * VALUES(I,'LSM')) =G= 40   ;
NUMSSM..      SUM( I, X(I) * VALUES(I,'SSM')) =G= 40   ;
NUMLAM..      SUM( I, X(I) * VALUES(I,'LAM')) =G= 40   ;
NUMSAM..      SUM( I, X(I) * VALUES(I,'SAM')) =G=  40   ;
NUMLASM..     SUM( I, X(I) * VALUES(I,'LASM')) =G= 30  ;
NUMSASM..     SUM( I, X(I) * VALUES(I,'SASM')) =G=  30  ;
NUMLGUNS..    SUM( I, X(I) * VALUES(I,'LGUNS')) =G= 13 ;
NUMSGUNS..    SUM( I , X(I) * VALUES(I,'SGUNS')) =G= 13 ;
BUDGET..      SUM( I, X(I) * VALUES(I,'C')) - DEVPOSB +  DEVNEGB
               =E=TOTBUDGET;
OBJFUN..       DEVPOSB =E= DEVIATION;

MODEL GP /ALL/;

SOLVE GP USING MIP MINIMIZING DEVIATION;
```

B. GAMS MIXED INTEGER PROBLEM OUTPUT

```
                     S O L V E    S U M M A R Y

   MODEL   GP              OBJECTIVE  DEVIATION
   TYPE    MIP             DIRECTION  MINIMIZE
   SOLVER  XA              FROM LINE  74

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE              1.1300

 RESOURCE USAGE, LIMIT          0.160    50000.000
 ITERATION COUNT, LIMIT         4        20000

No better solution than :     1.13000
                             Absolute                Relative
Actual distance              0.00000                 0.00000
Tolerances   (OPTCA)        0.00000   (OPTCR)        0.00000
```

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU NUMLSM	40.000	60.000	+INF	.
---- EQU NUMSSM	40.000	44.000	+INF	.
---- EQU NUMLAM	40.000	40.000	+INF	0.025
---- EQU NUMSAM	40.000	40.000	+INF	0.050
---- EQU NUMLASM	30.000	34.000	+INF	.
---- EQU NUMSASM	30.000	48.000	+INF	.
---- EQU NUMLGUNS	13.000	14.000	+INF	.
---- EQU NUMSGUNS	13.000	14.000	+INF	.
---- EQU BUDGET	2.800	2.800	2.800	-1.000
---- EQU OBJFUN	.	.	.	-1.000

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR DEVIATION	-INF	1.130	+INF	.
---- VAR DEVNEGB	.	.	+INF	1.000
---- VAR DEVPOSB	.	1.130	+INF	.

---- VAR X number of ships I to purchase

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	.	100.000	0.700
DDG	.	2.000	100.000	0.400
FFG	.	6.000	100.000	.
FSG	.	4.000	100.000	.
PBFG	.	1.000	100.000	0.130

APPENDIX E. GAMS GOAL PROGRAMMING FORMULATION AND OUTPUT FOR THREAT CONSTRAINED AND AHP PREFERRED MODEL

A. GAMS FORMULATION

```
SETS
   I  type of combat ships / CG      cruisers
                             DDG     guided missile destroyers
                             FFG     guided missile frigates
                             FSG     guided missile corvettes
                             PBFA    guided missile fast patrol boats  /
   P  parameters for each ship type
      / C       cost of ship type I in billions of dollars /
        LSM     number of the long  range surface to surface missile launchers
        SSM     number of the short  range surface to surface missiles launchers
        LAM     number of the long range surface to air missiles launchers
        SAM     number of the short range surface to air missiles launchers
        LASM    number of the long range anti submarine rockets or torpedoes
        SASM    number of the short range anti submarine rockets or torpedoes
        LGUNS   number of the long range  guns ( for aw and asuw )
        SGUNS   number of the short range guns ( for asuw and aaw ) /
   W  priority weights
      / CW      cost weights
        SW      speed weights
        AAWW    aaw  capability weights
        ASWW    asw  capability weights
        ASUWW   asuw capability weights
        AMPW    amphibious warfare capability weights
        CIWS    ciws capability weights
        FW      fuel consumption weights
        SCW     surveillance capability weights      /;
 TABLE
VALUES (I,P) parameter values for each ship type
              C   LSM  SSM  LAM  SAM  LASM  LSSM  LGUNS  GUNS
      CG     .9    16    0    8    0     8     6      2     2
      DDG    .6     8    0    8    0     8     3      2     2
      FFG    .3     4    4    4    4     0     6      1     0
      FSG    .2     4    4    0    4     0     6      1     2
      PBFA  .13     4    4    0    0     0     0      0     2
;
```

```
TABLE
     WEIGHTS (I,P) parameter values for each ship type

              CW    SW   AAW    ASWW    ASUWW    AMPW   CIWS   FW   SCW

      CG     .049  .132  .375   .238    .319     .388   .287  .089  .356
      DDG    .091  .152  .272   .331    .221     .270   .263  .169  .257
      FFG    .181  .169  .227   .297    .220     .210   .255  .200  .232
      FSG    .289  .220  .087   .099    .135     .091   .148  .226  .103
      PBFA   .390  .327  .039   .035    .106     .041   .047  .316  .051
;

PARAMETER
     NUM (I)     desired number of ship type I (unattainable)
          / CG        20
            DDG       20
            FFG       20
            FSG       20
            PBFA      20              /;

INTEGER VARIABLES
     X (I)            number of ships I to purchase
     DEVNEG (I)       negative deviation from desired number of ships of  type I
     DEVPOS (I)       positive deviation from desired number of ships of  type I   ;

FREE VARIABLE
     DEVIATION       deviation from objective function ;

EQUATIONS
      NLSMUP         number of lsm launchers needed to meet threat's surface power
                      (upper limit)
      NLSMLO         (lower limit)
      NSSMUP         number of ssm launchers needed to meet threat's surface
                      (upper limit)
      NSSMLO         (lower limit)
      NLAMUP         number of lam launchers needed to meet threat's air power
                      (upper   limit)
      NLAMLO         (lower limit)
      NSAMUP         number of sam launchers needed to meet threat's air power
                      (upper limit)
      NSAMLO         (lower limit)
      NLASMUP        number of lasm launchers needed to meet threat's submarines
                      (upper limit)
      NLASMLO        (lower limit)
      NSASMUP        number of sasm launchers needed to meet threat's submarines
                      (upper limit)
      NSASMLO        (lower limit)
```

```
NLGUNSUP    number of lguns needed to meet threat's power (upper limit)
NLGUNSLO    (lower limit)
NSGUNSUP    number of sguns needed to meet threat's power (upper limit)
NSGUNSLO    (lower limit)
NSHIPS(I)   desired number of ship type I (unattainable)
OBJFUN      objective function;

NLSMUP..    SUM( I, X(I) * VALUES(I,'LSM')) =L= 55   ;
NLSMLO..    SUM( I, X(I) * VALUES(I,'LSM')) =G= 30   ;
NSSMUP..    SUM( I, X(I) * VALUES(I,'SSM')) =L= 55   ;
NSSMLO..    SUM( I, X(I) * VALUES(I,'SSM')) =G= 30   ;
NLAMUP..    SUM( I, X(I) * VALUES(I,'LAM')) =L= 55   ;
NLAMLO..    SUM( I, X(I) * VALUES(I,'LAM')) =G= 30   ;
NSAMUP..    SUM( I, X(I) * VALUES(I,'SAM')) =L= 55   ;
NSAMLO..    SUM( I, X(I) * VALUES(I,'SAM')) =G= 30   ;
NLASMUP..   SUM( I, X(I) * VALUES(I,'LASM')) =L= 41   ;
NLASMLO..   SUM( I, X(I) * VALUES(I,'LASM')) =G= 23   ;
NSASMUP..   SUM( I, X(I) * VALUES(I,'SASM')) =L= 41   ;
NSASMLO..   SUM( I, X(I) * VALUES(I,'SASM')) =G= 23   ;
NLGUNSUP..  SUM( I, X(I) * VALUES(I,'LGUNS')) =L= 15   ;
NLGUNSLO..  SUM( I, X(I) * VALUES(I,'LGUNS')) =G= 11   ;
NSGUNSUP..  SUM( I , X(I) * VALUES(I,'SGUNS')) =L= 15   ;
NSGUNSLO..  SUM( I , X(I) * VALUES(I,'SGUNS')) =G= 11   ;
NSHIPS(I).. X(I) - DEVPOS(I) + DEVNEG(I) =E= NUM(I) ;
OBJECFUN..  DEVNEG( 'CG' ) * SUM(W,WEIGHTS('CG',W))
            + DEVNEG( 'DDG' ) * SUM(W,WEIGHTS('DDG',W))
            + DEVNEG( 'FFG' ) * SUM(W,WEIGHTS('FFG',W))
            + DEVNEG( 'FSG' ) * SUM(W,WEIGHTS('FSG',W))
            + DEVNEG( 'PBFA' ) * SUM(W,WEIGHTS('PBFA',W))
            =E= DEVIATION ;

MODEL GP /ALL/;

SOLVE GP USING MIP MINIMIZING DEVIATION;
```

B. GAMS MIXED INTEGER PROBLEM OUTPUT

```
              S O L V E     S U M M A R Y

   MODEL   GP              OBJECTIVE  DEVIATION
   TYPE    MIP             DIRECTION  MINIMIZE
   SOLVER  XA              FROM LINE  121

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
```

**** OBJECTIVE VALUE 157.2200

RESOURCE USAGE, LIMIT	0.330	50000.000
ITERATION COUNT, LIMIT	35	20000

No better solution than : 157.22000

	Absolute		Relative
Actual distance	0.00000		0.00000
Tolerances (OPTCA)	0.00000	(OPTCR)	0.00000

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU NLSMUP	-INF	52.000	55.000	.
---- EQU NLSMLO	30.000	52.000	+INF	.
---- EQU NSSMUP	-INF	52.000	55.000	.
---- EQU NSSMLO	30.000	52.000	+INF	.
---- EQU NLAMUP	-INF	32.000	55.000	.
---- EQU NLAMLO	30.000	32.000	+INF	.
---- EQU NSAMUP	-INF	40.000	55.000	.
---- EQU NSAMLO	30.000	40.000	+INF	.
---- EQU NLASMUP	-INF	24.000	41.000	.
---- EQU NLASMLO	23.000	24.000	+INF	.
---- EQU NSASMUP	-INF	36.000	41.000	.
---- EQU NSASMLO	23.000	36.000	+INF	.
---- EQU NLGUNSUP	-INF	10.000	15.000	.
---- EQU NLGUNSLO	10.000	10.000	+INF	.
---- EQU NSGUNSUP	-INF	10.000	15.000	.
---- EQU NSGUNSLO	10.000	10.000	+INF	.

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU OBJFUN	.	.	.	-1.000

---- VAR X number of ships I to purchase

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	.	100.000	-2.233
DDG	.	.	100.000	-2.026
FFG	.	8.000	100.000	EPS
FSG	.	2.000	100.000	-1.398
PBFA	.	3.000	100.000	-1.352

---- VAR DEVNEG negative deviation from desired number of ships of type I

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	20.000	100.000	.
DDG	.	20.000	100.000	.
FFG	.	12.000	100.000	1.991
FSG	.	18.000	100.000	.
PBFA	.	17.000	100.000	.

---- VAR DEVPOS positive deviation from desired number of ships of type I

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	.	100.000	2.233
DDG	.	.	100.000	2.026
FFG	.	.	100.000	.
FSG	.	.	100.000	1.398
PBFA	.	.	100.000	1.352

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR DEVIATION	-INF	157.220	+INF	.

APPENDIX F. GAMS QUADRATIC GOAL PROGRAMMING FORMULATION AND OUTPUT FOR BUDGET-CONSTRAINED AND AHP PREFERRED MIX MODEL

```
SETS
   I   type of combat ships / CG         cruisers
                              DDG        guided missile destroyers
                              FFG        guided missile frigates
                              FSG        guided missile corvettes
                              PBFA       guided missile fast patrol boats   /
   P   parameters for each ship type
       / C   cost of ship type I in billions of dollars /

 TABLE
     VALUES (I,P) parameter values for each ship type
                  C
        CG       0.9
        DDG      0.6
        FFG      0.3
        FSG      0.2
        PBFA     0.12                          ;

PARAMETER
     NUM (I)     desired number of ship type I (unattainable)
          / CG      20
            DDG     20
            FFG     20
            FSG     20
            PBFA    20                    /;

SCALARS
     TOTBUDGET       /2.6 /;

INTEGER VARIABLES

     X (I)       number of ships I to purchase
     DEVNEG (I)  negative deviation from desired number of ships of  type I
     DEVPOS (I)  positive deviation from desired number of ships of  type I   ;

FREE VARIABLE
     DEVIATION   deviation from objective function ;

EQUATIONS
     BUDGET      stay within budget
     NSHIPS(I)   desired number of ship type I (unattainable)
     OBJECFUN    objective function;

     BUDGET..   SUM(I, X(I) * VALUES(I,'C'))  =L= TOTBUDGET   ;
```

```
    NSHIPS(I).. X(I) - DEVPOS(I) + DEVNEG(I) =E= NUM(I) ;
    OBJECFUN..  1.9*(DEVNEG( 'CG' ) **2)+ 1.9*DEVNEG ('CG') +
            1.4*(DEVNEG( 'DDG' ) **2)+ 1.4*DEVNEG ('DDG')+
            1.1*(DEVNEG( 'FFG' ) **2)+ 1.1*DEVNEG ('FFG')+
            0.8*(DEVNEG( 'FSG' ) **2)+ 0.8*DEVNEG ('FSG')+
            0.5*(DEVNEG( 'PBFA' ) **2)+0.5*DEVNEG ('PBFA')
            =E= DEVIATION ;

MODEL GP /ALL/;
SOLVE GP USING RMINLP MINIMIZING DEVIATION;
```

B. GAMS QUADRATIC MIXED INTEGER PROBLEM OUTPUT

```
                S O L V E    S U M M A R Y

   MODEL   GP              OBJECTIVE  DEVIATION
   TYPE    MINLP           DIRECTION  MINIMIZE
   SOLVER  MINOS5            FROM LINE  77

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE            2049.9530

 RESOURCE USAGE, LIMIT          0.110     50000.000
 ITERATION COUNT, LIMIT         4         20000
 EVALUATION ERRORS              0         0

                  LOWER     LEVEL     UPPER    MARGINAL

---- EQU BUDGET        -INF      2.600     2.600  -131.469

                        LOWER     LEVEL     UPPER    MARGINAL

---- EQU OBJECFUN         .          .          .         -1.000

 ---- VAR X          number of ships I to purchase

             LOWER     LEVEL     UPPER    MARGINAL

CG             .          .       100.000      40.422
DDG            .          .       100.000      21.482
FFG            .        4.000     100.000       .
FSG            .        4.000     100.000      EPS
PBFA           .        5.000     100.000      EPS
```

---- VAR DEVNEG negative deviation from desired number of ships of type I

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	20.000	100.000	.
DDG	.	20.000	100.000	.
FFG	.	16.000	100.000	.
FSG	.	16.000	100.000	.
PBFA	.	15.000	100.000	.

---- VAR DEVPOS positive deviation from desired number of ships of type I

	LOWER	LEVEL	UPPER	MARGINAL
CG	.	.	100.000	77.900
DDG	.	.	100.000	57.400
FFG	.	.	100.000	39.441
FSG	.	.	100.000	26.294
PBFA	.	.	100.000	15.776

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR DEVIATION	-INF	2049.953	+INF	

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